Antonine Physics AS

Topic 6 Solid Materials

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is an electronic book which you can download. You can carry it in a portable drive and access it from your school's computers (if allowed) as well as your own at home.

This topic introduces you to the study of materials. Material science is an important discipline as it is vital that materials have the correct properties for the job in hand. The A380 Airbus is a massive machine with a mass of over 600 tonnes. It can only fly because the materials from which it is made are very light, but very strong. (Actually, many of these very expensive machines are being retired and even scrapped after a very short working life. They are too big for many airports, and they are extremely expensive to run.) Contrast this with a chipboard cupboard that is very heavy but very feeble.

Material scientists carry out a range of tests on new materials to see how they behave when stretched, squashed or twisted. They see what happens when they use extreme conditions (e.g. very hot or very cold).

I have included a tutorial on Fluid Materials. Yes, I know that it shouldn't be in a topic called Solid Materials. It is NOT on the AQA syllabus. It provides background material for a practical on terminal speed of a ball bearing in a viscous fluid. Some of it is on the Welsh Board and the OCR syllabus. If in doubt, ask your tutor.

Topic 6 Solid Materials

Tutorial 6.01 Material Properties

All Syllabi

Contents

6.011 Material Properties

It is important for a material to have the right properties for the job in hand. For example, glass is a stiff material (i.e. doesn't change shape easily), but it's very brittle. Therefore, it will not make a good structural material for a bridge. Stone is very strong in **compression** (i.e. when squashed) but is quite weak in **tension** (when pulled). Therefore, stone bridges are built to withstand high compressive forces. Old stone bridges were built to carry horses and carts (a mass of no more than 2500 kg) but today carry 40 tonne lorries (*Figure 1*).

Figure 1 A stone bridge

Notice how that on this bridge, there are steel straps on the right-hand side of the arch. Clearly the arch has deformed under tensile loads (pulling forces) and has started to crack. Steel is very **strong** when subjected to a **tensile** load and is strengthening the bridge. On the far left of the bridge, just behind the tree, you can see another steel frame that is holding the wing-walls together. These measures, along with a weight limit, will ensure the bridge will remain safe for many years.

Figure 2 The Mathematical Bridge in Cambridge (Photo by Chris Millar, Wikimedia Commons)

Wood and iron can be used to make bridges. There is a myth that Sir Isaac Newton designed a bridge (the Mathematical Bridge in *Figure 2*) over the River Cam in Cambridge that required no bolts or nails and built it himself. Actually, it was designed by William Etheridge and built by James Essex in 1749 (22 years after Newton's death). Sorry to ruin a good story.

Cast iron was widely used in bridges, although it can be brittle, causing bridge failure. At 19.15 on Sunday 28th December 1879 a train crossed the Tay Bridge during a severe storm. The bridge collapsed, plunging the train into the River Tay. Everyone on board was killed. Poor design work and slovenly construction contributed to the disaster. A new and more substantial bridge was built alongside, and much of the old bridge was re-used in the new. It was opened in 1887 and has been in use since. So, the girders were not that bad...

6.012 Some Definitions

Property	***Definition***
Strength	How much force is needed to break something. Not always a fair comparison. Something that is thick will be stronger than a thin section. A fairer test is the **breaking stress**.
Breaking Stress	Breaking stress = breaking force ÷ area Force is applied at 90 ° to the area.
Stiffness	How difficult it is to change the shape of the object. If we load wires of the same length and diameter with the same tension, the stiffest is the one that stretches the least.
Brittle	Stiff, but not strong
Elastic	Ability of a material to regain its original shape after it is distorted.
Plastic	A material that does NOT regain its original shape after it is distorted.
Flexible	Can bend easily without breaking or deforming.
Ductile	Can be drawn out into wires
Tough	How well a material resists breaking when forces are applied.
Malleable	Can be hammered into shape.
Alloy	A mixture of metals

6.013 Density

Density is defined as

mass per unit volume.

Density, mass, and volume are linked by a simple relationship:

$$\rho = \frac{m}{V} \quad \ldots\ldots\ldots\ldots\ldots \quad \textit{Equation 1}$$

The strange looking letter ρ is *rho*, a Greek letter 'r'. It is the Physics code for density.

SI Units for density are **kg m^{-3}**. In some texts, you will find some densities given in g cm^{-3}. It is important that you use the **SI units** otherwise formulae will not work. To convert you will need the following conversion:

1 g cm^{-3} = 1000 kg m^{-3}

Lengths must be in metres.

1 mm = 1 × 10^{-3} m
1 cm = 1 × 10^{-2} m

Mass must be in kilograms.

1 g = 1 × 10^{-3} kg

Watch out for these bear traps.

Also, the physics code ρ has nothing to do with resistivity.

6.014 Measuring Density

We can measure density quite simply by weighing the sample on a top pan balance. Most top-pan balances measure in grams. We need to convert from grams to kilograms by dividing by 1000.

If the object is regular, the volume is easily worked out by measuring the dimensions (*Figure 3*) and applying a formula, e.g.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

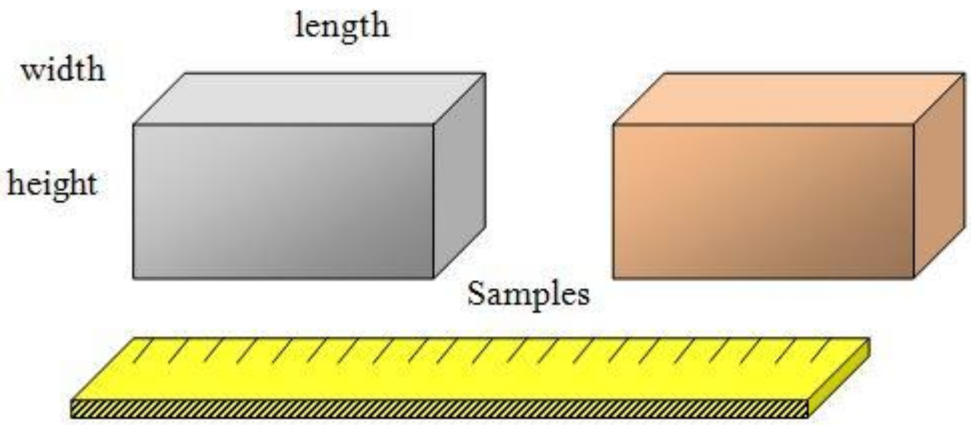

Figure 3 Measuring the volume of a rectangular sample

The volume will be in cm^3. Again, we need to convert cm^3 to m^3. $1\ cm^3 = 1 \times 10^{-6}\ m^3$.

Here are some other regular objects (*Figure 4*):

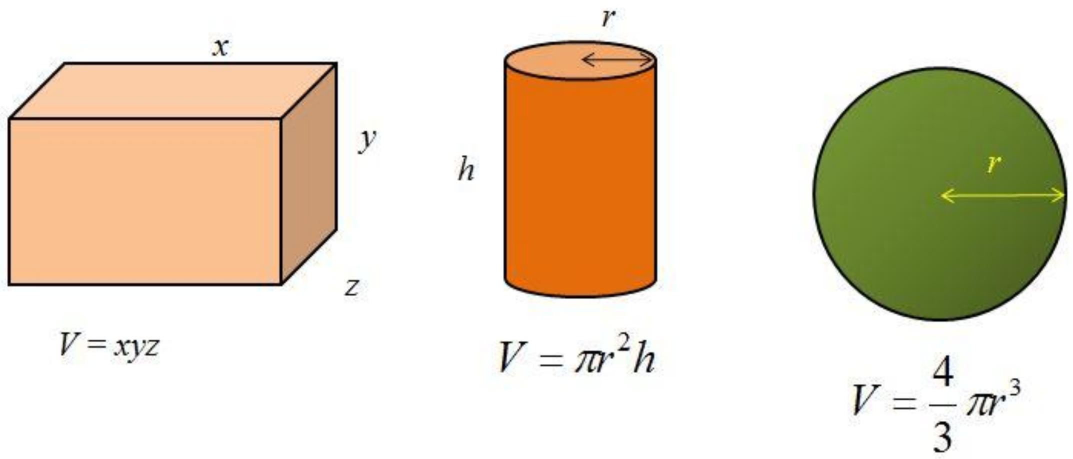

Figure 4 Volumes of regular sold objects

You do remember how to work out these volumes, don't you?

An irregular object can be lowered into a **eureka** (or **displacement**) can. The volume of displaced water can be measured (*Figure 5*).

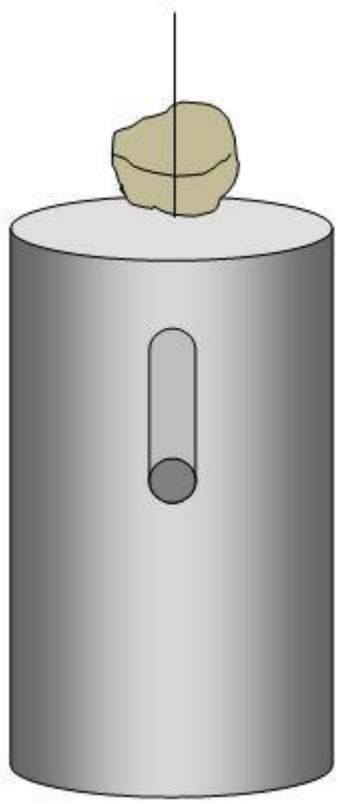

Figure 5 Measuring the volume of an irregular object using a eureka can.

The link to the Engineering Toolbox site will take you to some typical **densities** of metals and alloys. Click HERE.

6.015 Crystalline Materials

Metals are crystalline, which means they have a regular and long-range structure. A typical arrangement is like this (*Figure 6*).

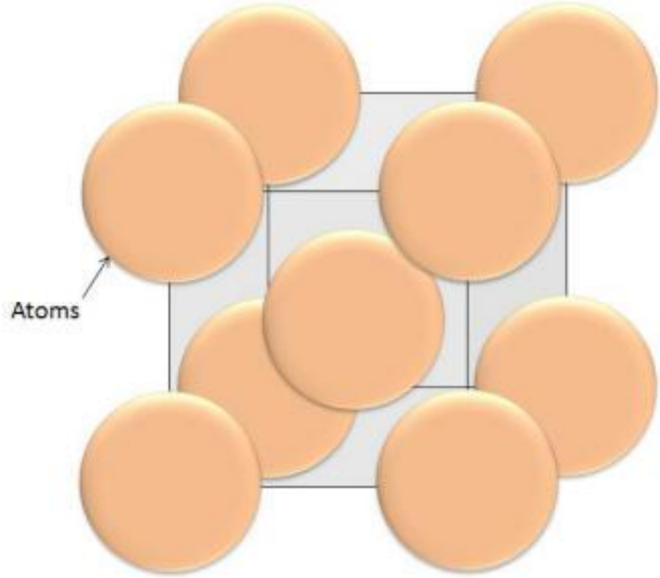

Figure 6 A cubic crystal structure

This arrangement is called **cubic**, as eight of the atoms are at the corners of a cube. There is also one in the middle.

If we expand the arrangement, we see layers like this (*Figure 7*).

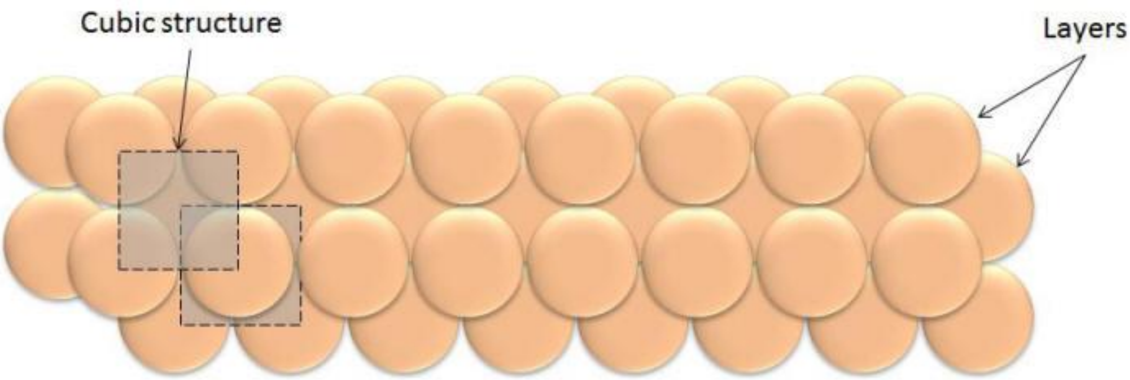

Figure 7 A long-range structure based on cubic crystal subunits.

When sufficient tensile (pulling) stresses applied, the layers can ride over each other like this (*Figure 8*).

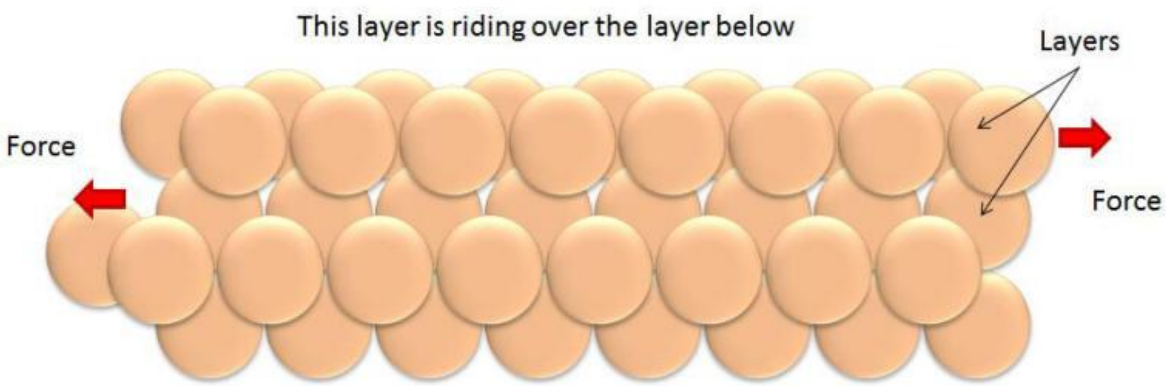

Figure 8 Applying a tensile force to a long-range cubic crystal structure.

To give this (*Figure 9*).

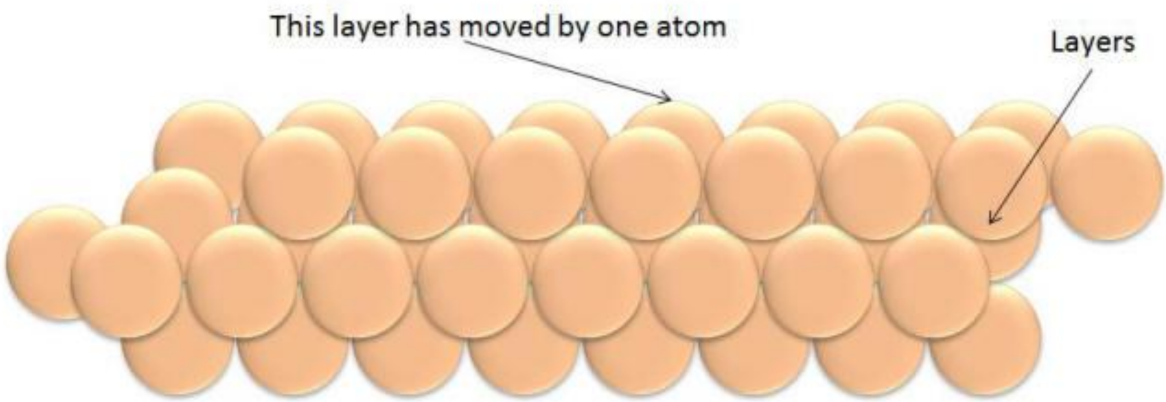

Figure 9 Deformation of a cubic crystalline structure.

When this happens, the crystal has been **permanently deformed**. This can be useful in ductile materials and malleable materials.

Pure materials tend to be less strong than **alloys**. Also, the crystalline structure, although long range, is not infinite. Stress points occur along crystal boundaries, which can weaken the structure.

6.016 Alloys

Engineers and scientists need to evaluate the properties of the materials they intend to use. **Alloys** are mixtures of elements, usually metals. These may have physical properties that are quite different from the pure metals. For example, pure iron is a rather weak metal. If it contains a certain proportion of carbon, it makes steel, which is much stronger. If there is more carbon, then the alloy is rather brittle, and can shatter. However, cast iron machines well, and is resistant to rust. The idea is shown below (*Figure 10*).

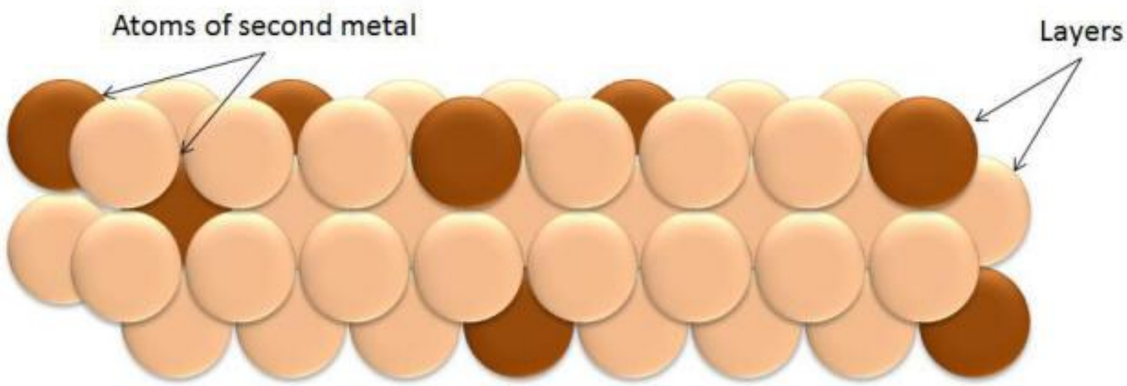

Figure 10 Alloy

Alloys can be tailored to special requirements. Duralumin has been used for many years in the aviation industry. It consists of:

- 4.4% copper,
- 1.5% magnesium,
- 0.6% manganese,
- 93.5% aluminium.

If we know the proportions of each metal in the alloy we can work out the density. We can find the densities of each metal in a data-book:

Metal	Density (kg m^{-3})
Aluminium	2712
Copper	8930
Magnesium	1738
Manganese	7210

Worked example What is the density of duralumin?
Answer Let us assume that the volume is going to be 1 m^3. These are the relative proportions. Use mass = density × volume: • 4.4% copper = 0.044 ×8930 kg = 393 kg. • 1.5% magnesium = 0.015 × 1738 kg = 26.1 kg • 0.6% manganese = 0.006 × 7210 kg = 42.3 kg • 93.5% aluminium = 0.935 × 2712 kg = 2536 kg Total mass in 1 m^3 = 393 kg + 26.1 kg + 42.3 kg + 2536 kg = 2997.4 Density = 3000 kg m^{-3}.

As well as being very low density, duralumin is also very hard and durable.

6.017 Amorphous Materials

Materials like glass do not have a crystal structure. They are called **amorphous**. They are more like liquids in that they have short-range groups of molecules with bonds between the small groups. If the bonds between the small groups did not exist, the material would be a liquid. If such materials are heated sufficiently, they become a melt.

Such materials tend to be rather brittle, and shatter is subjected to a sudden stress. When a surface nick is subjected to the stress, cracks propagate from the nick, and the material breaks (*Figure 11*).

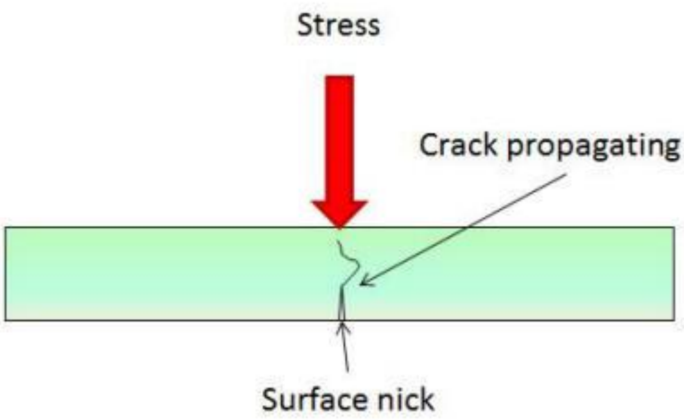

Figure 11 Propagation of a crack in glass

When cutting glass, glaziers score a line along a piece of glass with a diamond scriber. Then they split the glass along the score. It breaks cleanly (so they say). Some glass has wire strands bedded in it to strengthen the glass. Also, if the glass does break, the wire holds it together.

6.018 Polymers

Polymers are made from subunits called **monomers**, which are lined up in long chains. The monomer shown in *Figure 12* is **ethene** (or ethylene). It is the building block of many polymers.

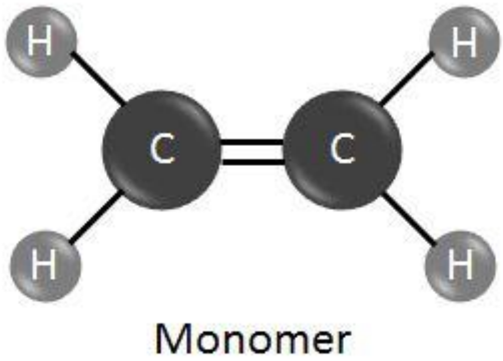

Figure 12 An ethene monomer

The monomer is subjected to a **polymerisation** reaction that breaks the double bond to make a long chain like this (*Figure 13*)

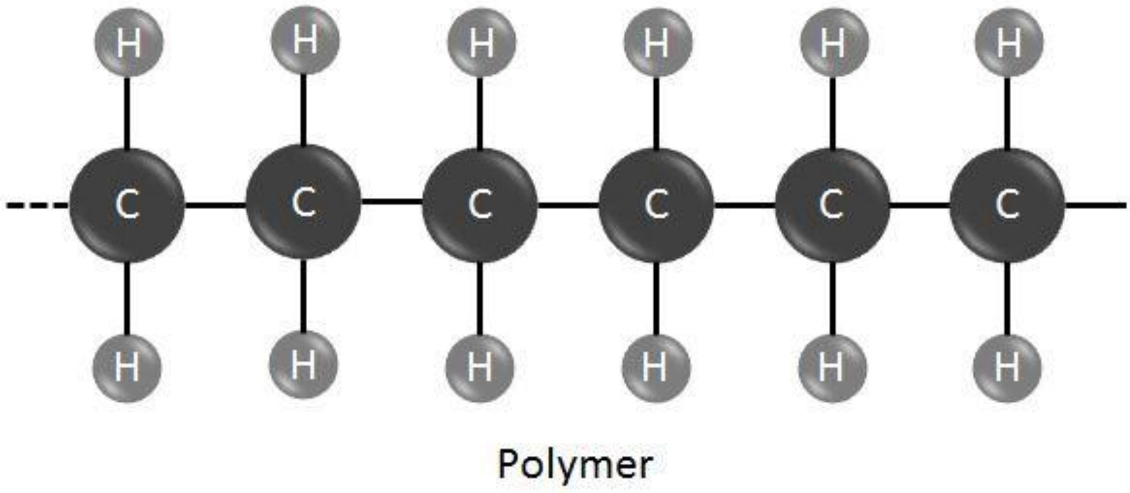

Figure 13 A polyethene chain

The reaction to make this happen occurs under conditions of high temperature and pressure. The precise mechanism is not part of these notes. If you want to know, ask a chemist.

Weak forces between the chains hold the chains together. The chains can easily slide over each other, so polythene can be easily deformed. You can try it for yourself, using the polythene straps that hold four drinks tins together. Pull them so that plastic deformation happens. You will feel that the polythene gets hot.

Many polymer chains use an **ethene backbone**. They have side chains of various different kinds, for example, benzene rings on styrene molecules (*Figure 14*).

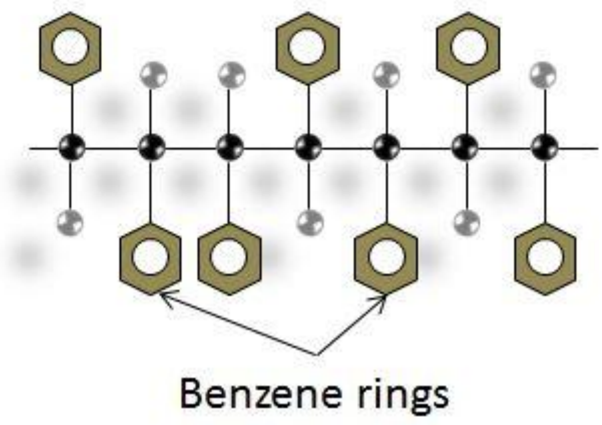

Figure 14 Benzene rings make a polymer chain more rigid

This results in improved rigidity of the polymer.

If the polymer is heated, it melts. It then can be injected into a mould. It will then solidify and retain the shape of the mould. Such a polymer is called a **thermoplastic**. Polymers like polystyrene or polyvinyl chloride can be dissolved using a solvent. **Adhesives** (glues) for these kinds of polymers are based on these solvents.

There is another kind of polymer called a **thermoset**. There are two or more molecules that need to react to make a polymer. The compounds are placed into the mould and are allowed to set or cure. Once the material has hardened, it will not melt on heating; it will decompose. One such example is Bakelite. When it gets hot it gives off triethylamine, which stinks of fish that is none-too-fresh.

There is much concern about the **pollution** caused by plastics. When they were first used, they were thought to be wonder materials as they were unreactive and cheap to manufacture. Nobody ever anticipated the problems that would arise. Many such materials break down into smaller and smaller pieces, eventually to form **microplastics** that can be ingested by wildlife. They can result in damage to the health of entire ecosystems. At the time of writing, attempts are being made to restrict the amount of plastics and the types of chemicals with which they can be treated. Agreement has, so far, been elusive.

Biological polymers (**proteins**) are made of a variety of amino acid monomers. There are 20 amino acids that build up a huge range of proteins that make our bodies function. Some of these proteins have catalytic functions (**enzymes**). They work fastest at 37 °C. At temperatures above about 40 °C, the structure of the polymers alters permanently. The proteins are **denatured**.

6.019 Using materials

Aeroplanes are made out of materials that have very low density. They have to have a low mass, and every effort is made to ensure that the machine does not carry anything that is not needed. Therefore, the machine can carry useful load that enables it to earn money. They are made from aluminium (density 2700 kg m^{-3}), or an alloy of aluminium and titanium (very expensive). Some aircraft are made of carbon fibre, which is very strong, but has a very low density.

A large aircraft like a Jumbo jet (Boeing 747), if made of steel, would barely waddle about the aerodrome. It could not take off. Using low density materials, giant freight aircraft such as the Antonov 225 (*Figure 15*) can be built.

Figure 15 Antonov A-225 (Flight simulator image)

Originally designed to take the Russian space shuttle on its back, this brute can take a 100-tonne railway locomotive easily in its hold. A sad note: Only one of these magnificent machines was ever built and it was wrecked in the squalid war that Russia launched against Ukraine in 2022. As I write this, the abysmal war continues as a war of attrition. There is hope that the Antonov 225 will be rebuilt and put back into service.

Boeing was proposing to build something even bigger, the Pelican, with a mass of 1000 tonnes empty, with a payload of 800 tonnes, and an appetite for fuel to match. Later, the project was quietly abandoned over doubts concerning the prospects of its commercial success.

6.020 Composite Materials

A typical **composite** material is shown below (*Figure 16*).

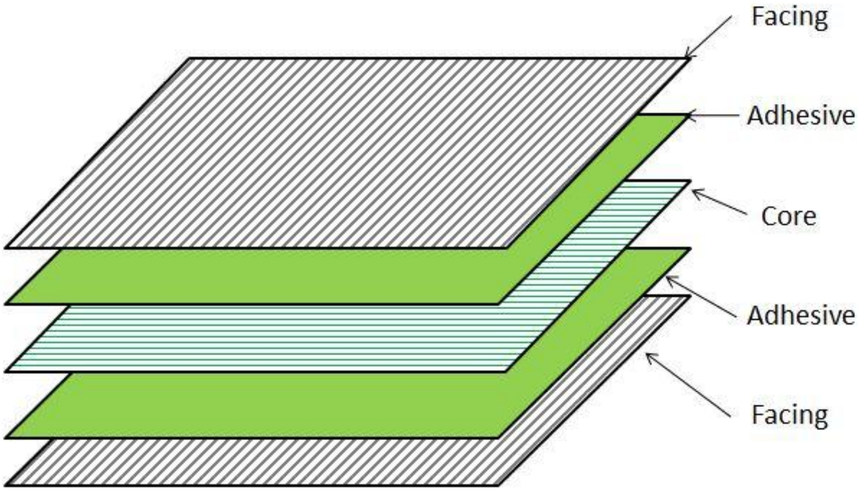

Figure 16 A composite material

The core can be a **honeycomb structure** which is particularly strong.

Composite materials are not confined to the aerospace industry. Composite materials such as **plywood** are made up in layers of wood in which the grains cross at 90 degrees to each other. Plywood is often found in shelving, covered with a veneer of fine wood. Plywood is stronger than shelves made of the equivalent thickness of normal wood.

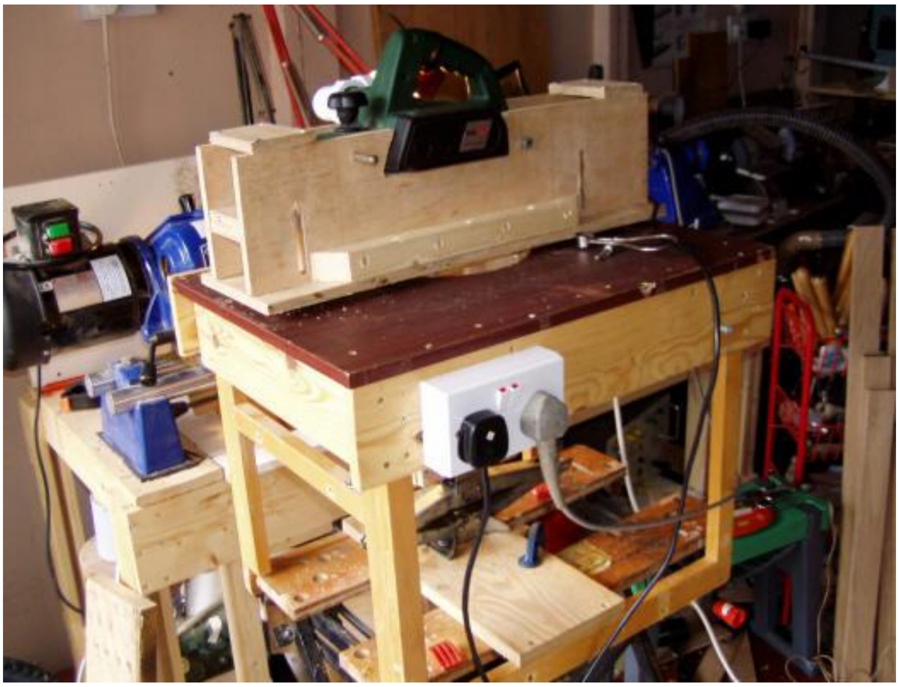

Figure 17 A home-made thicknesser

The picture above (*Figure 17*) shows a home-made **thicknesser** (an electric planer that can machine wood to a chosen thickness). The planer machine is mounted in a plywood jig. (I made the machine, but the results were disappointing. Later it broke.)

Another composite material found in the furniture industry is chipboard. Unlike plywood, it is very heavy and very feeble. But it's cheap.

Concrete is very strong in **compression** (squashing) but very feeble in **tension** (stretching). **Reinforcing wires** are inserted to make the reinforce concrete stronger in tension. Many concrete structures are **pre-stressed**, meaning that the wires are under tension when the concrete is moulded. The stressed wires, when released, pull the concrete together, making it even stronger.

The composite materials above are rigid. However flexible composites are widely available, for example Kevlar. Police officers wear body protection made from Kevlar to prevent injuries from attempts to stab them. Thicker Kevlar is used in bullet-proof jackets. This dog (*Figure* 18) is bullet and stab-proof in his Kevlar jacket.

Figure 18 A police dog in a Kevlar jacket (Source not known)

Tutorial 6.01 Questions

6.01.1

Give one example of a material that is:

Stiff:

Brittle:

Plastic:

Elastic and strong:

6.01.2

What are the following densities in kg m^{-3}?

a) 1.29 g cm^{-3}

b) 7.6 g cm^{-3}

c) 19.6 g cm^{-3}

6.01.3

What is the volume of 100 g of Mercury of which the density is 13 600 kg m^{-3}?

6.01.4

When is a plastic not a plastic?

6.01.5

An empty paint tin has a diameter of 0.150 m and a height of 0.120 m, and a mass of 0.22 kg. It is then filled with paint to a depth of 7mm *from the top*. Now the paint tin has a total mass of 6.50 kg. Calculate:

a. The mass of the paint.

b. The volume of the paint.

c. The density of the paint.

Give your answer to an appropriate number of significant figures.

6.01.6

An alloy tube of volume 1.8×10^{-4} m^3 consists of 60 % aluminium and 40 % magnesium by volume. Calculate:

a. The mass of:

i. Aluminium

ii. Magnesium in the tube.

b. The density of the alloy.

Density of aluminium is 2700 kg m^{-3} and magnesium 1700 kg m^{-3}.

Tutorial 6.02 Hooke's Law

All Syllabi

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6.021 Springs

If we load a spring, we find that the **extension** (code e) or stretch is **proportional** to the **force** (code F). If we double the force, we double the stretch. This is called **Hooke's Law**.

$$F \propto e$$

$$\Rightarrow F = ke \quad \text{............ Equation 2}$$

The constant of proportionality is called the **spring constant** (or **force constant**) and is measured in **newtons per metre** (N m^{-1}).

We can also treat a system which is under compression in a similar way. The e term is called the **compression** rather than the extension.

Remember to convert the extension into metres and the load into Newtons before working out the spring constant.

In the AQA syllabus, the extension is given the code Δl, i.e. the change in length. In some text books, you may see it given as x. In these notes, you will see all three used.

We can plot this as a graph (*Figure 19*).

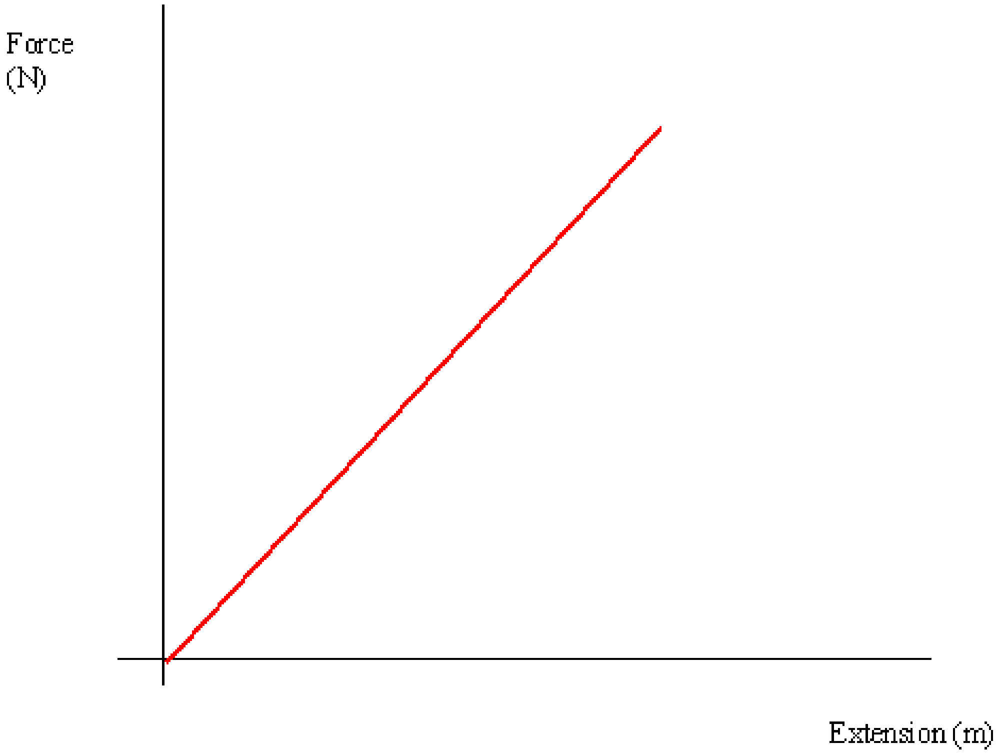


Figure 19 Force-extension graph

We can see that the graph is a straight line and that the **gradient** gives us the spring constant. That is why we have the extension on the horizontal axis. We say that force is **directly proportional** to the extension:

- The line is a straight line...
- ...that goes through the origin...
- ...and has a positive gradient.

The same is true if we apply a squashing force, a **compression** force.

6.022 Springs in series and parallel

It may seem strange to bring in terms more familiar in electric circuits, but we can have springs arranged in series and parallel. The picture below (*Figure 20*) shows a single spring, then two springs in **series**.

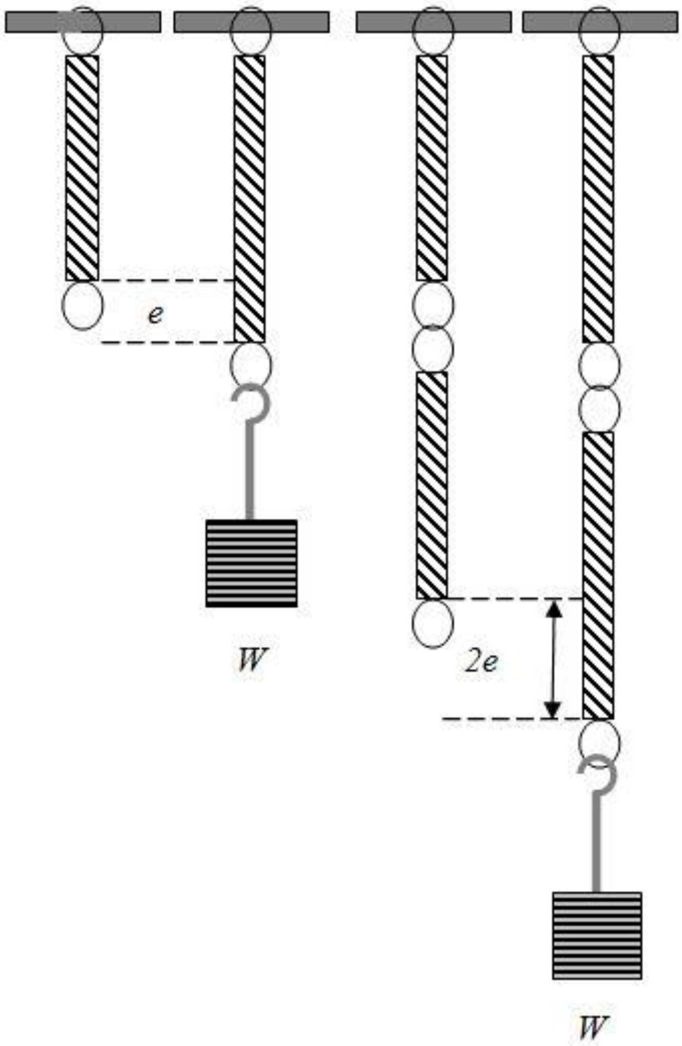


Figure 20 Springs in series

If we load the first spring with a weight W, we see that it extends by e. Now we attach a second identical spring in series, and put on the weight W. The same force acts through each spring, so each spring stretches by e. Therefore, the total stretch is $2e$.

Since $k = F/e$, we can easily see that the spring constant halves.

The picture below (*Figure 21*) shows the same two springs in **parallel**:

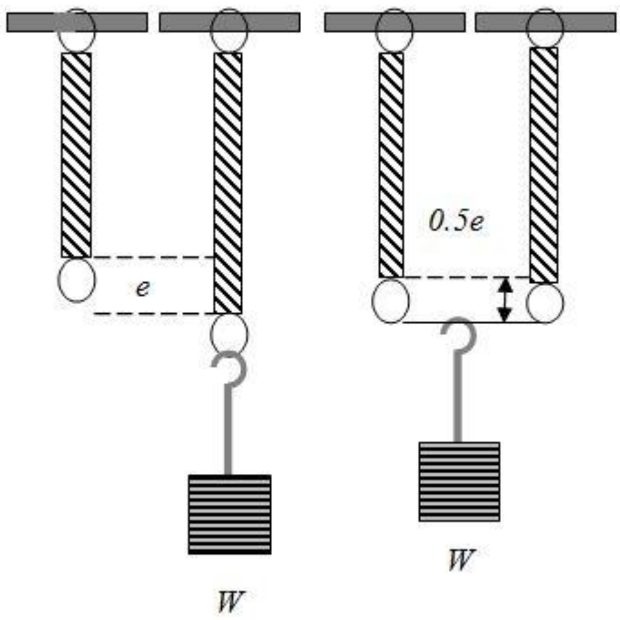


Figure 21 Parallel springs

The springs are identical to the ones before. This time the weight W is shared out equally between the two. Since each spring has $0.5\ W$ acting on each one, the stretch is $0.5\ e$. The spring constant of the parallel springs is therefore:

$$k = W \div 0.5e = 2k$$ .... *Equation 3*.

Cars can be modelled as a block of mass m, placed on 4 springs each of spring constant k (*Figure 22*).

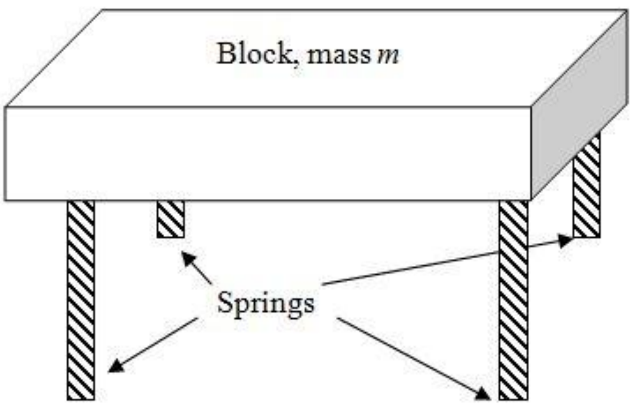


Figure 22 Modelling a car as a block mounted on 4 springs

6.023 Elastic Strain Energy

The energy is the **area** under the **force-extension** graph. How do we achieve this result?

If we stretch the spring by a tiny amount *dl*, we do a tiny job of work:

$$\delta W = \delta l \times F$$ .... *Equation 4*

This is shown by the little rectangle (*Figure 23*).

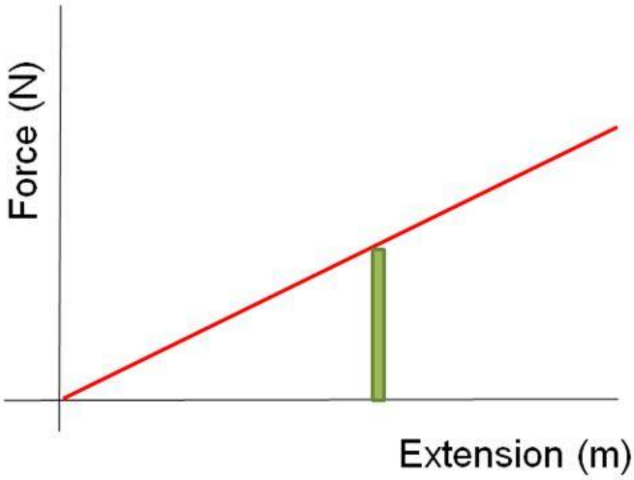


Figure 23 A tiny job of work shown on a force-extension graph

Do it again, we get another little rectangle (*Figure 24*).

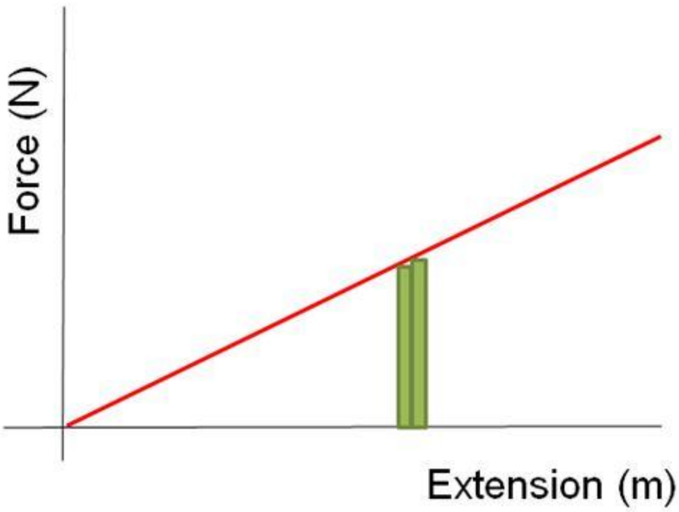


Figure 24 And again....

Now fill in all the little rectangles (*Figure 25*).

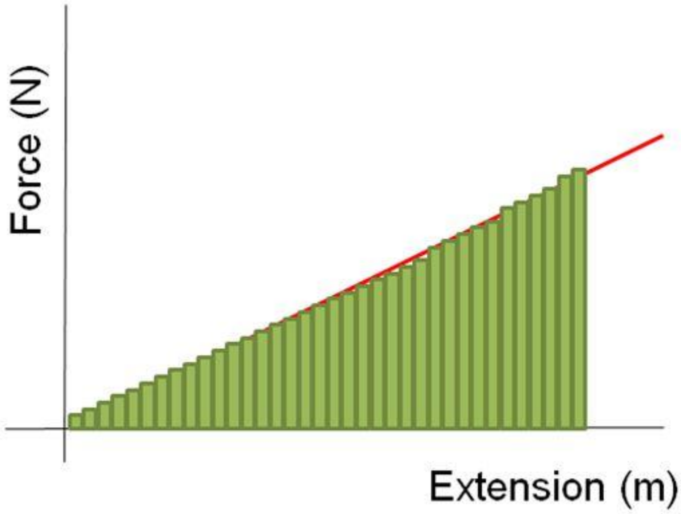


Figure 25 Lots of little rectangles

All the little rectangles give area under the graph to give *Equation 5*.

$$E = \frac{1}{2} F \Delta l$$

.... *Equation 5*

A neater graph is shown here (*Figure 26*).

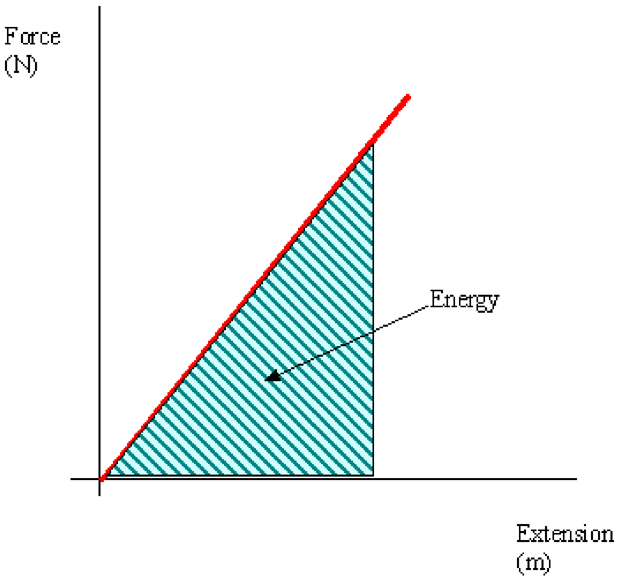


Figure 26 Energy is the area under the force-extension graph

So, we can use this result to say:

$$E = \frac{1}{2}F\Delta l$$

.... *Equation 6*

This result can also be obtained using the process of **integration**, which is part of the branch of mathematics called **calculus**. The expression will use e as the code for extension:

$$E = \int_0^e ke\,de = \frac{1}{2}ke^2$$

.... *Equation 7*

You are not expected to know this for AS, although you will be if you study Physics at University. All we need to say here is that the energy is the area of the triangle.

We can derive two further expressions from this result:

$$E = \frac{1}{2}k(\Delta l)^2$$

.... *Equation 8*

and

$$E = \frac{1}{2}\frac{F^2}{k}$$

.... *Equation 9*

6.024 Hysteresis

When we stretch the spring, we have to do a job of work. If we release the spring, we can recover that energy, which is called the **elastic strain energy**. Ideally, we recover all of it but in reality, a certain amount is lost as heat. This lost energy is called **hysteresis**. This is shown on the graph in the yellow triangle (*Figure 27*).

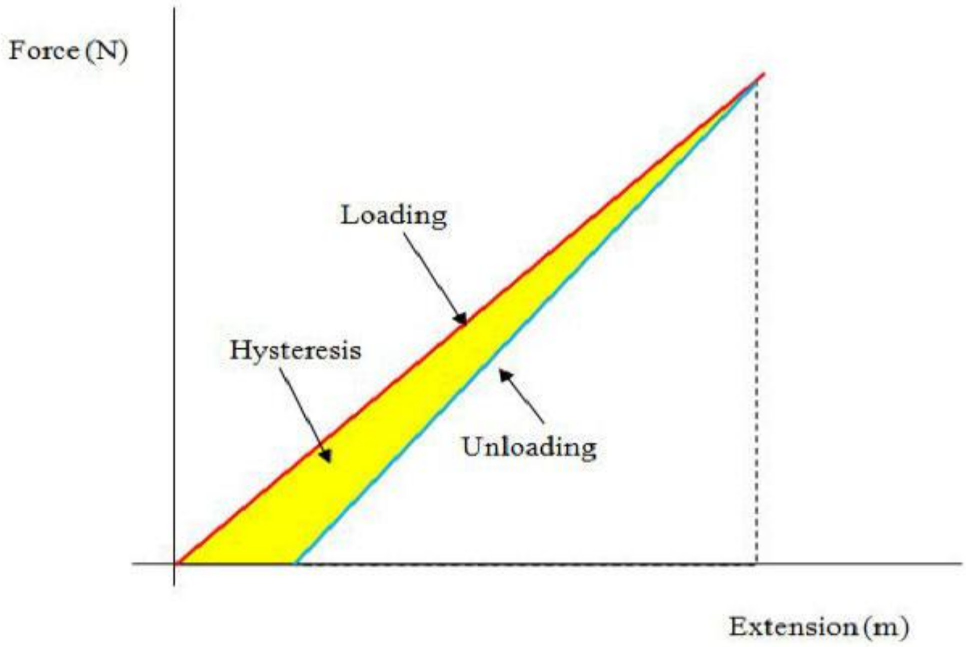


Figure 27 Showing hysteresis even when Hooke's Law is obeyed

From all hysteresis graphs, we should note:

- The lost energy is the shaded area between the loading and unloading lines.
- You would work out the energy lost by counting the squares in the shaded region.
- The smaller the squares counted, the less the uncertainty.

Hysteresis, please, not hysteria.

6.025 Elastomers

A **rubber band** is an example of a material that does not obey Hooke's Law. The force extension graph looks like this (*Figure 28*).

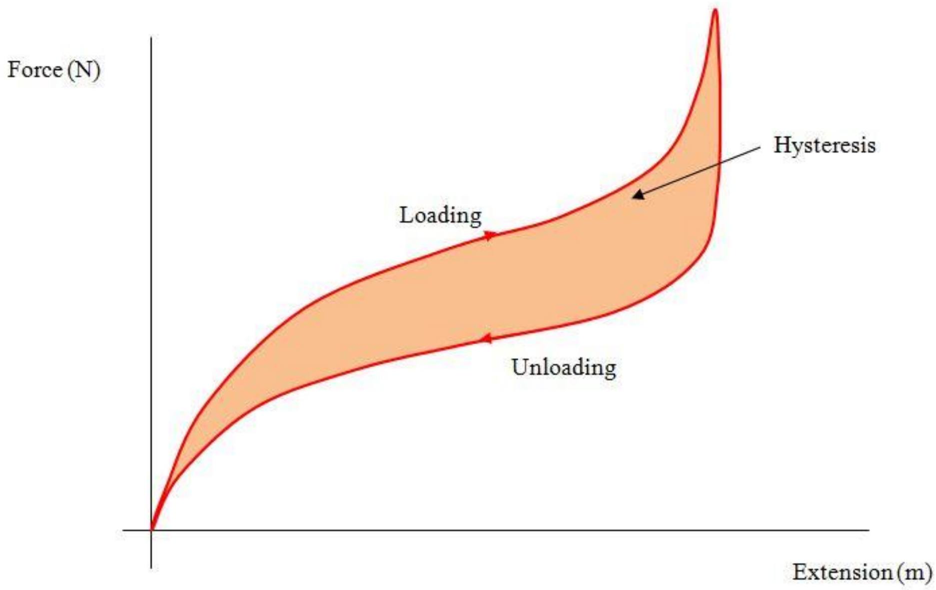


Figure 28 Force-extension graph for a rubber band

Rubber is an example of an **elastomer**. It is made up of long polymer chains that are tangled up with cross-links (**disulphide bridges**) between the chains. The chains with no load are rather like spaghetti. When the load is small, a large extension is seen because the chains are simply untangling (*Figure 29*).

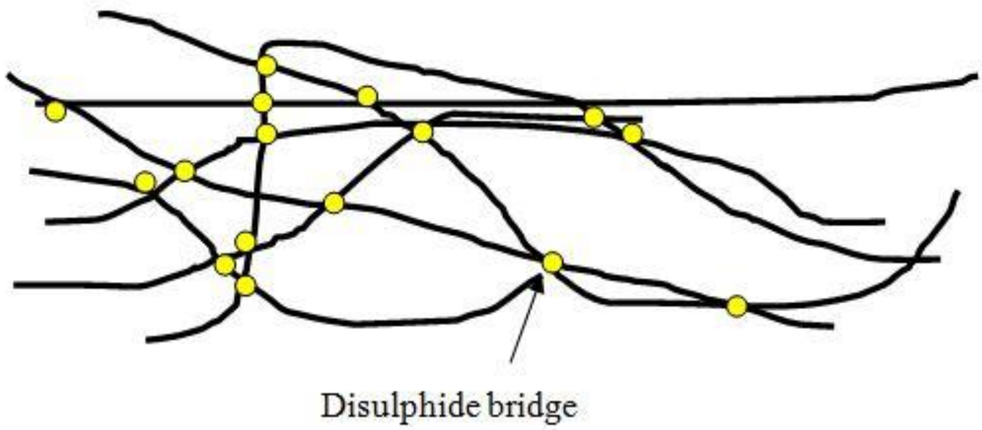


Figure 29 Chains in rubber are tangled up

Once the chains are untangled, then the bonds between atoms are stretched. This requires a greater force with a small extension (*Figure 30*).

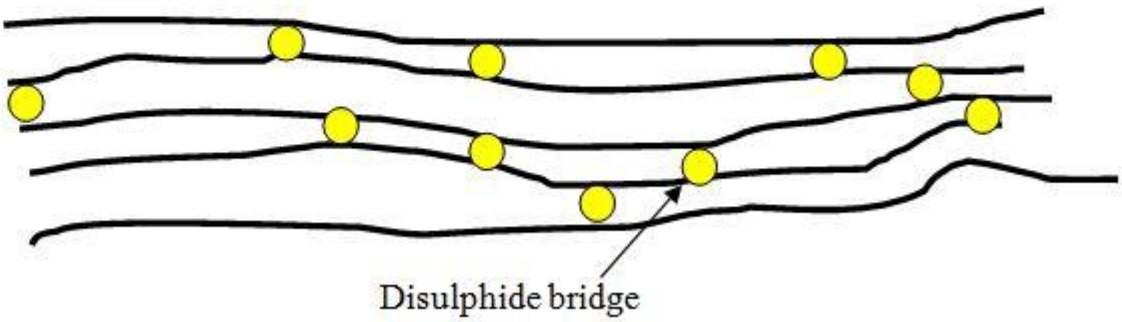


Figure 30 Rubber chains are stretched.

We can see from the graph that there is a large **hysteresis**. This is because work has to be done to break cross-links as the elastomer is stretched. When the elastomer is relaxed, the cross-links reform, and give out heat as they do so.

Natural rubber is quite soft and flexible as there are relative few disulphide bridges. Hard rubber is **vulcanised**, which means that more disulphide bridges are added.

6.026 Energy Changes in Springs

When a spring is stretched (in **tension**) or squashed (in **compression**), it stores energy as **elastic potential energy**. The energy can then be released to do a job of work, for example, to lift a mass against gravity (*Figure 31*).

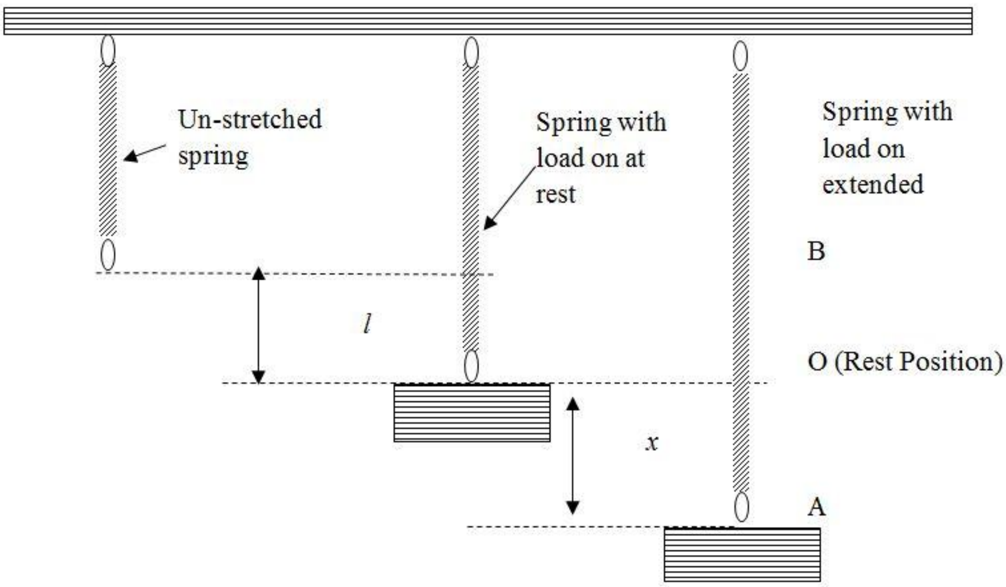


Figure 31 Elastic potential energy stored in a spring

The **extension** of a spring is directly **proportional** to the **force** (Hooke's Law). Consider a mass, m, put onto a spring of spring constant k so that so that it stretches by an extension l.

The energy changes are as follows:

- When stretched, the spring has **elastic potential** energy.
- When released, the spring converts that energy to **kinetic** energy, doing work by lifting the mass against the weight.
- The **inertia** of the mass enables the mass to pass the **rest** position, the position the mass would stay at if it were hanging freely.
- The kinetic energy is converted to **gravitational potential energy**.

The force on the spring = mg, and the stretching tension = kl.

Therefore

$$mg = kl \text{ .... } \textit{Equation 10}$$

Suppose the spring is pulled down by a distance x below the rest position. Now the stretching force becomes:

$$k\,(l + x) \text{ .... } \textit{Equation 11}$$

This is also the tension in the spring acting upwards.

So, the restoring force, F_{up}, is given by:

$$F_{up} = k\,(l + x) - mg \text{ .... } \textit{Equation 12}$$

This is because mg is the weight, which always acts **downwards**.

Since $kl = mg$, we can write:

$$F_{up} = kl + kx - kl = kx \text{ .... } \textit{Equation 13}$$

The height to which the mass will bounce is also x, the displacement by which the mass was pulled down.

The total energy is:

$$E_p = \frac{1}{2}kx^2$$ *............... Equation 14*

The **total energy** in the system remains the same. But it is converted from **potential** to **kinetic** and back again.

When released, kinetic energy is formed.

$$E_k = \frac{1}{2}mv^2$$ *............. Equation 15*

If we consider the speed, we can write:

$$kx^2 = mv^2$$ *.............. Equation 16*

Therefore, the speed is directly proportional to the extension.

We can show the energy interchange in this graph (*Figure 32*).

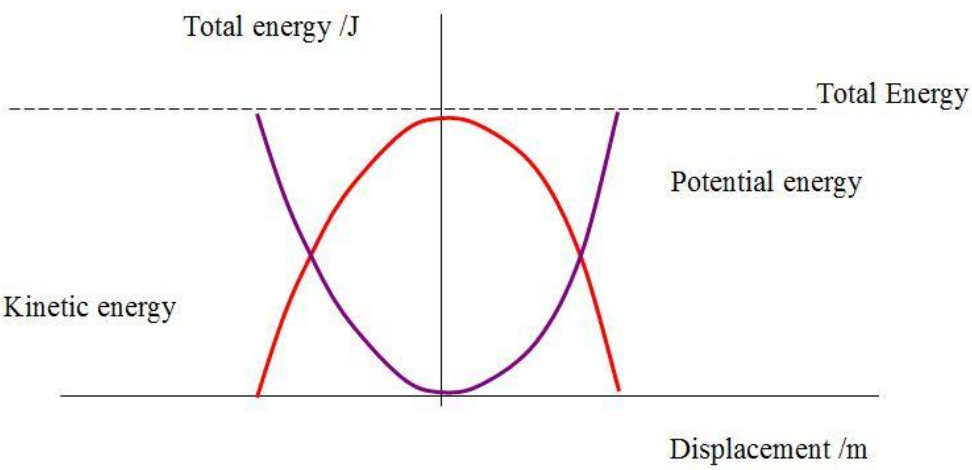


Figure 32 Energy in a bouncing spring

The shape of this graph is **sinusoidal**, because the mass-spring system **oscillates** (bounces) in **simple harmonic motion**. You will study this in the A-level year.

Like everything in Physics, it isn't perfect. Energy is lost as **heat**, so the amplitude of the bounce **decreases**. The energy lost can be shown to be a **constant fraction** of the total.

Tutorial 6.02 Questions

6.02.1

When a 500 g mass is placed on a spring, it stretches by 12 cm.

What is its spring constant in N m^{-1}?

6.02.2

When a 500 g mass is placed on a spring, it stretches by 12 cm. A second identical spring is now placed below the first.

a. How far do the two springs in series stretch?

b. What is the new spring constant in N m^{-1}?

6.02.3

When a 500 g mass is placed on a spring, it stretches by 12 cm. A second identical spring is now placed alongside the first.

a. How far do the two springs in parallel stretch?

b. What is the new spring constant in N m^{-1}?

6.02.4

A car of mass 1600 kg is placed on four identical springs. Each spring is seen to be compressed by a distance of 5 cm. What is the spring constant of each spring?

Use $g = 9.8$ N kg^{-1}

6.02.5

Show that $E = \frac{1}{2} ke^2$.

6.02.6

Use a similar method to show that $E = \frac{1}{2} F^2/k$

6.02.7

A car of mass 1600 kg is placed on four identical springs. Each spring is seen to be compressed by a distance of 5 cm. What is the energy in each spring?

Use g = 9.8 N kg^{-1}

6.02.8

A spring has a 300 g mass added to it. It has a spring constant of 35 N m^{-1}.

a. Calculate the amount by which it is stretched if the mass hangs freely.

b. The spring is then displaced a further 10 mm. Calculated the force applied.

c. Calculate the energy in the stretched spring.

Use g = 9.8 N kg^{-1}

Give your answers to an appropriate number of significant figures.

Tutorial 6.03 Stress, Strain, and the Young Modulus	
All Syllabi	
Contents	
6.031 Stress and Strain	6.032 Stress-Strain Curves
6.033 Young Modulus	6.034 Strain Energy per Unit Volume
6.035 Measuring Stress and Strain	6.036 Searle's Method

6.031 Stress and Strain

Wires obey **Hooke's Law** just like a spring. This is because bonds between atoms stretch just like springs (*Figure 33*).

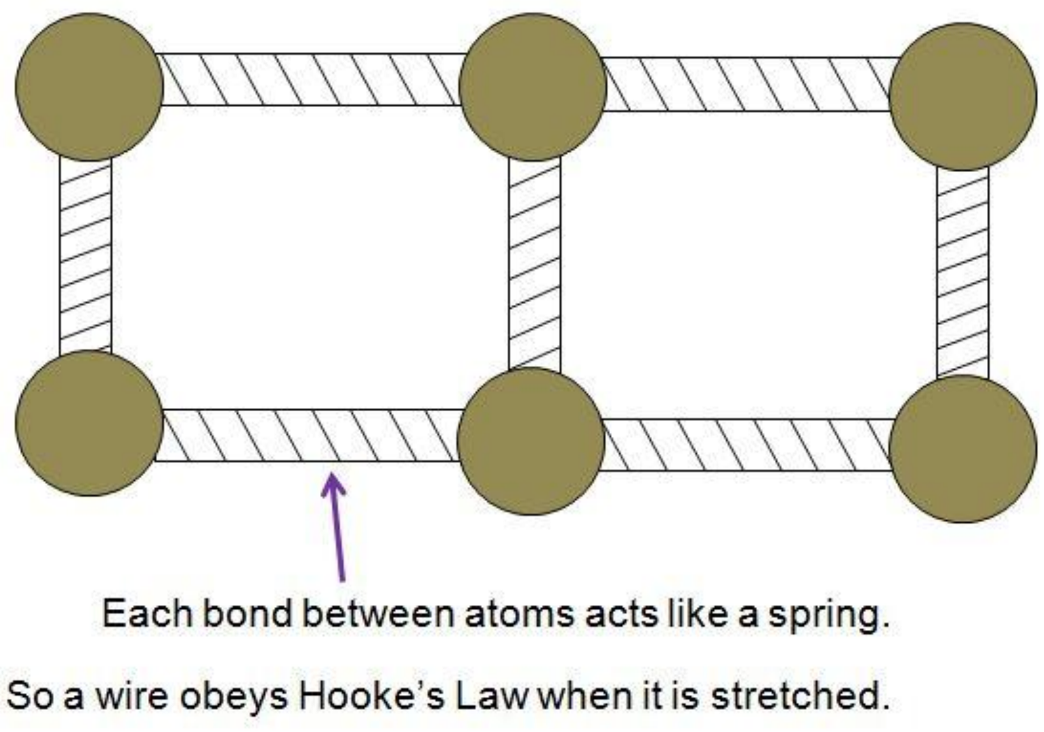


Figure 33 Bond between atoms stretch just like springs

If we stretch a wire, the amount it stretches by depends on:

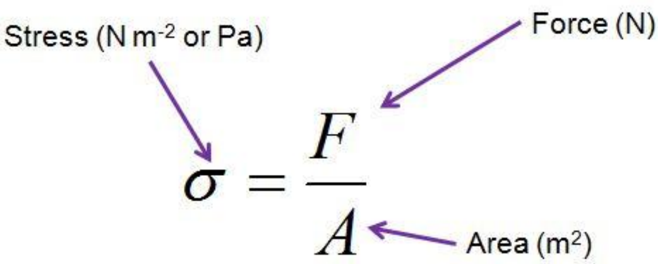
- its length
- its diameter
- the material it's made of.

If we have two of the same material and length but of different thicknesses, it is clear that the thicker wire will stretch less for a given load. We make this a fair test by using the term **tensile stress** which is defined as **the tension per unit area normal to that area**. The term **normal** means at 90° to the area.

We can also talk of the **compression** force per unit area, i.e. the **pressure**.

$$\text{Stress} = \frac{\text{Load (N)}}{\text{area (m}^2)} = \frac{F}{A}$$

In some text books you may see stress given the code σ (sigma, a Greek letter 's'). Therefore:



.... *Equation 17*

You will have met the expression F/A before. It is, of course, **pressure**, which implies a squashing force. A stretching force gives an expression of the same kind. Units are **newtons per square metre** (N m^{-2}) or **Pascal** (Pa).

1 Pa = 1 N m^{-2}

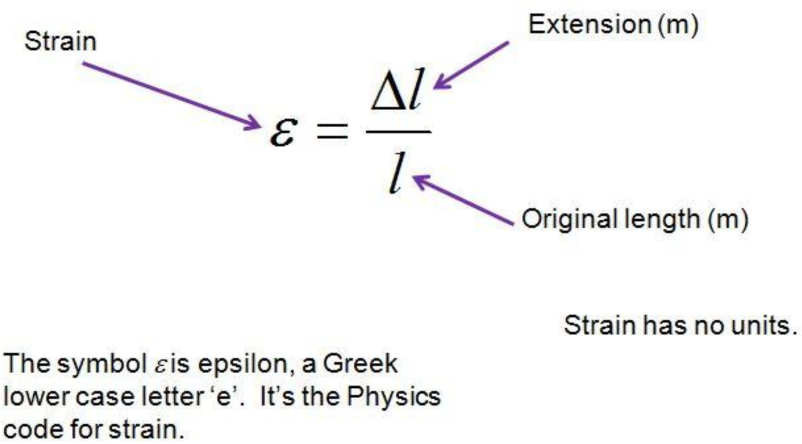
You must always convert areas to **square metres** for this equation to work. Remember that radii will often be given in mm or cm. This is a common bear trap.

- 1 mm^2 = 1 x 10^{-6} m^2
- Therefore, if you get an area of 10^{-2} m^2, you probably have forgotten to do the conversion to square metres.

If we have a wire of the same material and the same diameter, it doesn't take a genius to see that the wire will stretch more for a given load if it is longer. To take this into account, we express the extension as a ratio of the original length. We call this the **tensile strain** which we define as **the extension per unit length**.

$$\text{Strain} = \frac{\text{extension (m)}}{\text{original length (m)}}$$

There are no units for strain; it's just a number. It can sometimes be expressed as a percentage.



.... *Equation 18*

You will find that the same is true for when we compress a material.

6.032 Stress-Strain Curves

Stress-strain graphs are really a development of force-extension graphs, simply taking into account the factors needed to ensure a fair test. A typical stress-strain graph looks like this (*Figure 34*).

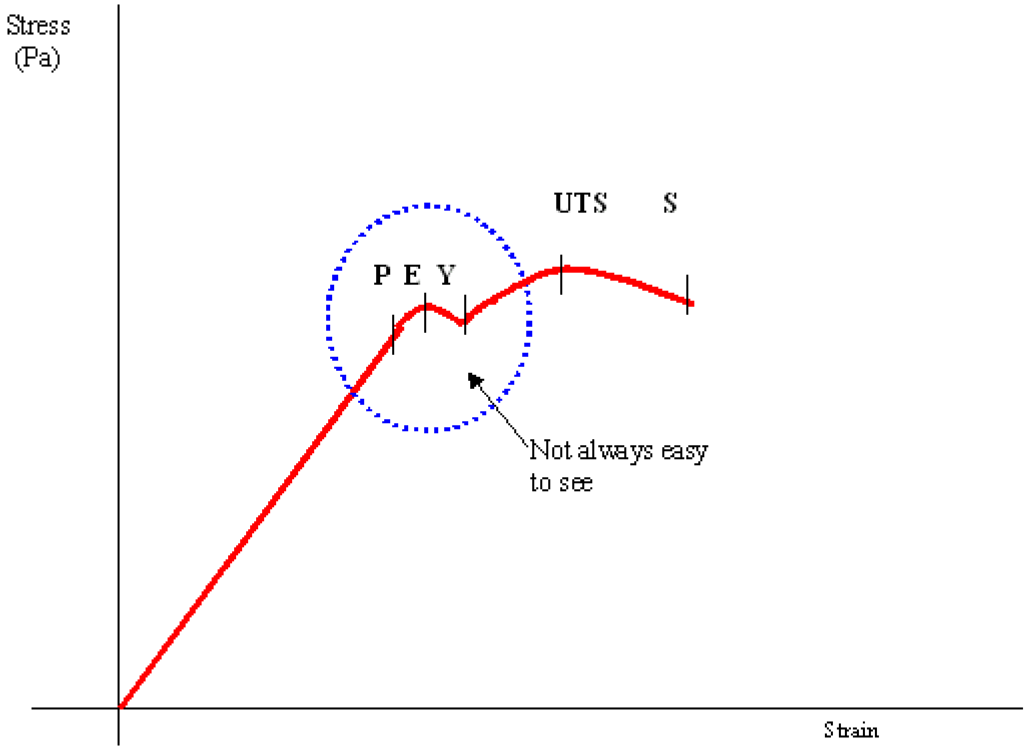


Figure 34 A stress-strain graph

We can describe the details of the graph as:

- **P** is the **limit of proportionality**, where the linear relationship between stress and strain finishes.
- **E** is the **elastic limit**. Below the elastic limit, the wire will return to its original shape.
- **Y** is the **yield point**, where **plastic deformation** begins. A large increase in strain is seen for a small increase in stress.
- **UTS** is the **ultimate tensile stress**, the maximum stress that is applied to a wire without its snapping. It is sometimes called the **breaking stress**. Notice that beyond the UTS, the force required to snap the wire is less.
- **S** is the point where the wire snaps.

We can draw stress-strain graphs of materials that show other properties (*Figure 35*).

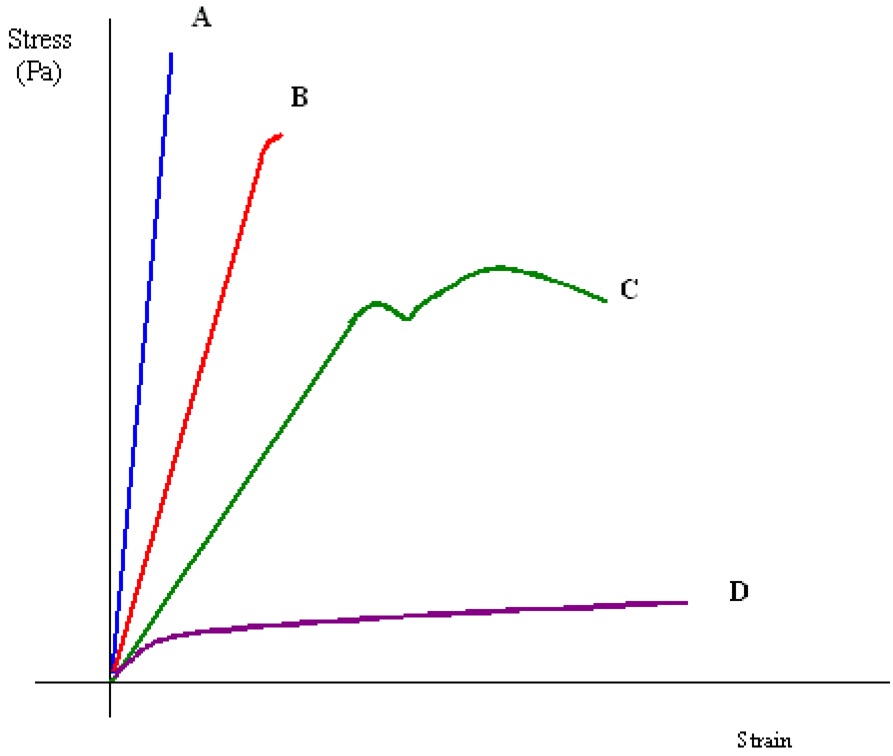


Figure 35 Stress-strain graphs for different material properties

- Curve **A** shows a **brittle** material. This material is also strong because there is little strain for a high stress. The fracture of a brittle material is sudden and catastrophic, with little or no plastic deformation. Brittle materials crack under tension and the stress increases around the cracks. Cracks propagate less under compression.
- Curve **B** is a **strong** material which is not ductile. Steel wires stretch very little and break suddenly. There can be a lot of elastic strain energy in a steel wire under tension, and it will "whiplash" if it breaks. The ends are razor sharp and such a failure is very dangerous indeed.
- Curve **C** is a **ductile** material
- Curve **D** is a **plastic** material. Notice a very large strain for a small stress. The material will not go back to its original length.

6.033 The Young Modulus

The Young Modulus is named after the British polymath Thomas Young (1773 - 1829). (A polymath is someone who knows everything.) The term modulus means a *little measurement*. I hate to spoil a good story (again), but it was actually Leonhard Euler (1707 – 1783) who worked it out. Young's Modulus experiments were performed by Giordano Ricatti in 1782. Young reported his findings in 1807.

The **Young Modulus** is defined as

the ratio of the tensile stress and the tensile strain.

So, we can write:

$$\text{Young modulus} = \frac{\text{tensile stress}}{\text{tensile strain}}$$

We know that:

$$\text{tensile stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A}$$

and that

$$\text{tensile strain} = \frac{\text{extension}}{\text{original length}} = \frac{\Delta l}{l}$$

Young Modulus has the physics code E, so we can write:

Young's Modulus (Pa) Stress (Pa)

$$E = \frac{\sigma}{\varepsilon}$$

Strain

.... *Equation 19*

which becomes:

$$E = \frac{Fl}{A\Delta l}$$

.... *Equation 20*

Units for the Young Modulus are **Pascals** (Pa) or **newtons per square metre** ($N\ m^{-2}$).

The Young Modulus describes **pulling forces**.

We can link the Young Modulus to a stress strain graph (*Figure 36*).

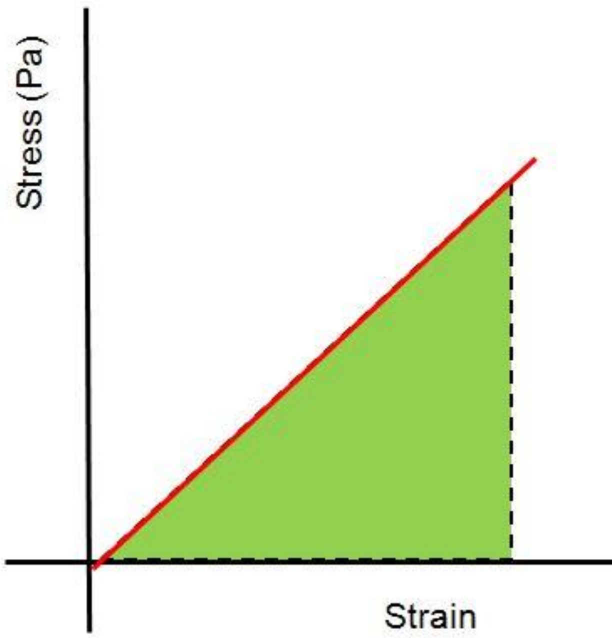


Figure 36 Youn's modulus is the gradient of a stress-strain graph

The Young Modulus is the **gradient** of the stress-strain graph for the region that obeys Hooke's Law. This is why we have the stress on the vertical axis when we would expect the stress to be on the horizontal axis.

6.034 Strain Energy per Unit Volume

The **area** under the stress strain graph is the **strain energy per unit volume** (joules per metre^3).

Strain energy per unit volume = 1/2 stress x strain.

The units arise because stress is in $N\ m^{-2}$ and strain is $m\ m^{-1}$ (NOTE: This unit here is not "millimetres to the minus one", but **metres per metre** which mean **no units**).

$N\ m^{-2} \times m\ m^{-1} = N\ m\ m^{-3}$. N m is joules, hence $J\ m^{-3}$

Area is the strain energy per unit volume. So, we can write this equation:

Area = ½ × stress × strain

In Physics code:

$$E' = \frac{1}{2}\frac{F\Delta l}{Al}$$

.... *Equation 21*

The term E' is pronounced "E-prime" or "E-dashed" and is being used as the code for the elastic strain energy per unit volume.

Al is area × length = volume

6.035 Measuring Stress and Strain

We can measure the Young Modulus doing a simple experiment like this (*Figure 37*).

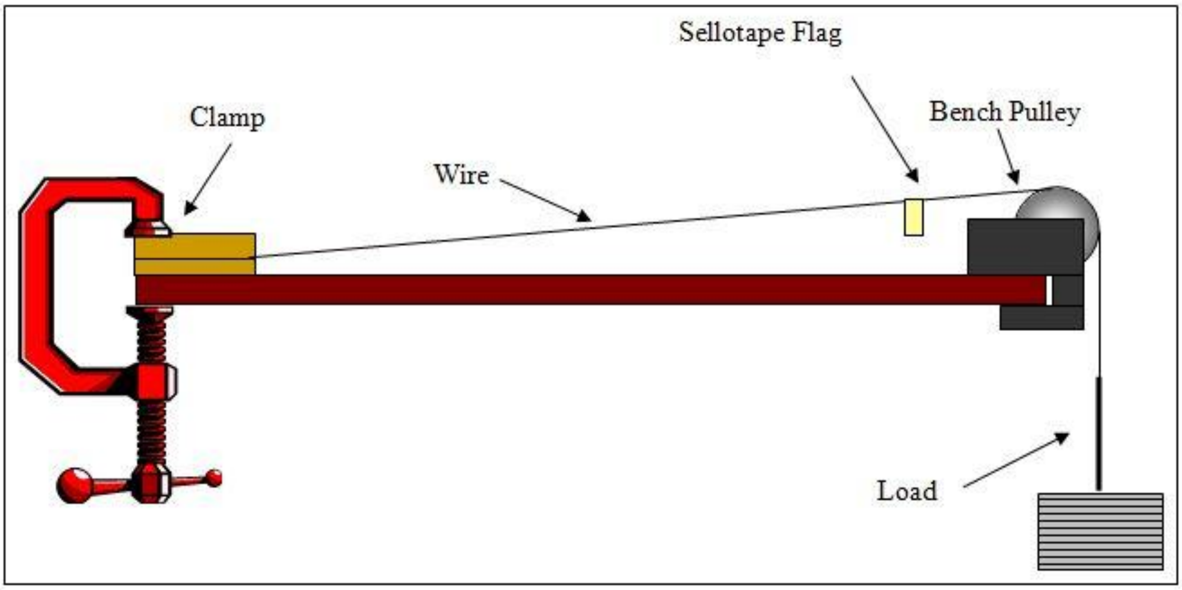


Figure 37 A simple way to measure Young's Modulus

We need to measure the diameter of the wire using a **micrometer**. A video tutorial in how to do this is in one of the links. Then we measure the extension as we increase the load, recording the observations. We need to convert the force into **stress**, and extension into **strain**. From that we plot a stress-strain graph, using the gradient to calculate the Young Modulus.

You need to have a couple of slotted masses on the wire to pull it taut. This will be your zero-reference point. Suppose you need 0.3 kg to pull the wire taut. Then you add 0.1 kg to make the total load 0.4 kg. The load you note in your results is **0.98 N**.

A useful tip is to place a sheet of graph paper underneath. Mark the position of the flag at your zero load, then mark the position under load.

You need to do at least three readings for each load and take an average.

The value we get for E is often rather low. This is because the wire might suffer the following defects:

- It might not be of uniform diameter. A smaller diameter will cause a greater stress.
- The crystal structure of the wire may be degraded by the wire drawing process. The wire is made by pulling the metal through a small hole called a **die**.

There are also many uncertainties. The greatest uncertainty comes in measuring the diameter. You will need to use a micrometer screw gauge like the one shown below (*Figure 38*).

Figure 38 A micrometer screw gauge (Source not known)

You need to do at least three readings along the length of the wire that you are using and take an average.

Your tutor will show you how to use it. There is an excellent tutorial on how to do this HERE.

This experiment is a **required practical** for A-level syllabuses in England.

6.036 Searle's Method

This is an alternative method that uses apparatus like this (*Figure 39*). You may have seen it hanging from the ceiling at the back of the physics lab at your school or college.

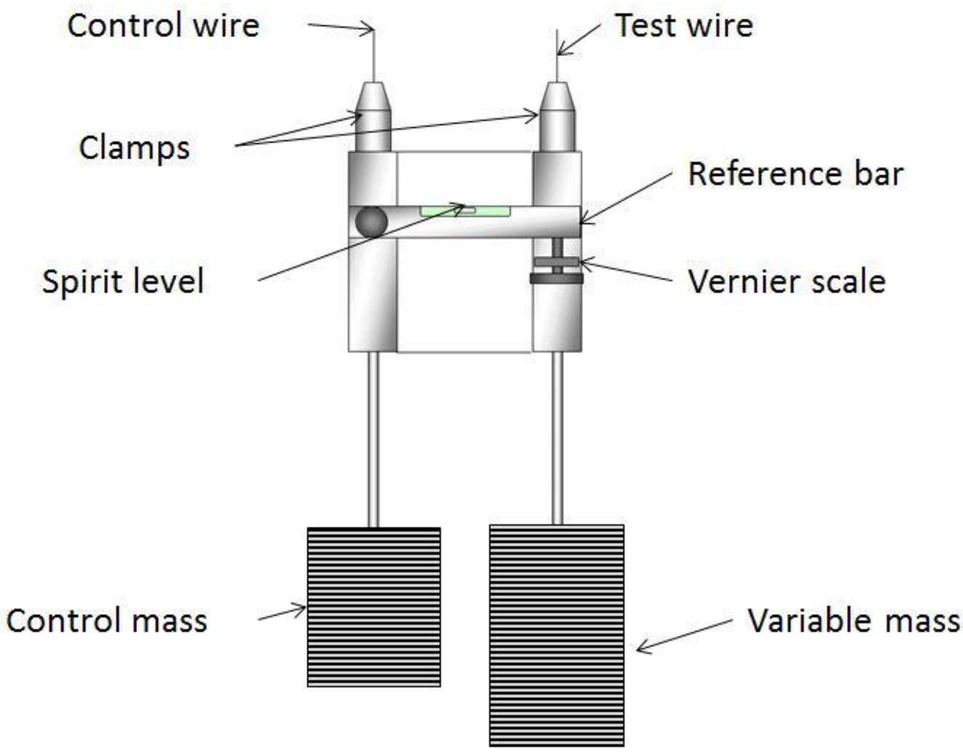


Figure 39 Searle's Apparatus (By Ewen at English Wikibooks, CC BY-SA 2.5, https://commons.wikimedia.org/w/index.php?curid=61793896 (Adapted))

This is how such an experiment is carried out:

- There are two wires of the same material, same diameter, and the same length. The control wire is held taught by a constant control mass which is not altered. It acts as a reference.
- The diameter needs to be measured with a micrometer.
- The variable mass is placed on the test wire. Initially it is the same mass as the control mass. This is the zero point.
- The reference bar is levelled off using the spirit level, and the vernier scale is set to zero.
- Then a mass is added to variable mass.
- The masses need to be converted to weights (g = 9.8 N kg^{-1})
- The vernier scale is adjusted, so that the reference bar is level.

- Then the reading is taken.

The advantages of this method are:

- The wires are long, so the strain is more easily measured.
- Both wires undergo the same thermal expansion, so the results are not affected by variations in the temperature in the lab.

The disadvantages are:

- The ceiling mounting needs to be secure. Should the control mass fall on your foot, you will know about it.
- The extensions are small, so there is more uncertainty.
- There is also uncertainty in measuring the diameter.

Tutorial 6.03 Questions

6.03.1

Find the area of a wire of diameter 0.75 mm in m^2.

6.03.2

What is the strain of a 1.5 m wire that stretches by 2 mm if a load is applied?

6.03.3

A wire made of a particular material is loaded with a load of 500 N. The diameter of the wire is 1.0 mm. The length of the wire is 2.5 m, and it stretches 8.0 mm when under load. What is the Young Modulus of this material?

Give you answer to an appropriate number of significant figures.

6.03.4

What is the elastic strain energy per unit volume for the wire in question 6.03.3?

Tutorial 6.04 Fluid Materials

Extension. Welsh Board and OC R

Contents

I have included this material as an extension. It provides background theory to the core practical measuring terminal speed of a ball bearing falling through glycerine.

Much of it is not on the AS or A-level syllabi for the examination. It will be studied if Physics is taken at a level beyond A-level, so it's worth knowing about it. However, do not spend your time on it unless you are confident with the content of whatever syllabus you are taking.

6.041 Fluids

A **fluid** material is one that adopts the shape of its container. It cannot resist any shear forces that are applied to it. It can be a liquid or a gas. The general properties of a fluid are:

- It cannot resist permanent deformation.
- It flows.

However, there are significant differences between fluids, depending on the phase of the matter:

- Liquids form a free surface that is not made by the container. This is on the top surface of the liquid. Liquids have a surface tension.
- Liquids are almost impossible to compress.
- Gases are not restricted by any top surface. Nor is there any surface tension.
- Gases can be compressed easily.

Some apparently solid materials are actually very viscous (gooey) fluids. Glass is one such example (*Figure 40*).

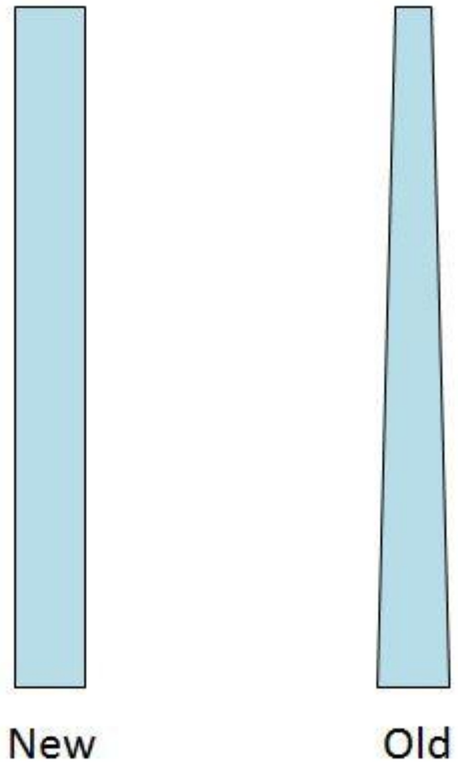


Figure 40 Glass is a very viscous fluid.

Other examples include silicone putty. You can roll it into a ball. Then leave it a while and it flattens out into a disc.

An **ideal fluid** is **incompressible**, has **zero viscosity** and has **zero shear stress**. An ideal fluid is **hypothetical**. Water is an almost ideal fluid. It is incompressible and has a low viscosity. Gases are NOT ideal fluids. This is because they are compressible.

This tutorial consider the behaviour of fluids that are **stationary**. In some text books, the situations are called **fluid statics**.

6.042 Pressure in a Fluid

You will have encountered pressure before. It is defined as:

perpendicular force per unit area

The equation is:

$$p = \frac{F}{A}$$

.... *Equation 22*

[p - pressure (N m^{-2}); F - force (N); A - area (m^{2}).]

The units for pressure are **Pascals** (Pa) or **Newton per square metre** (N m^{-2}) where:

1 Pa = 1 N m^{-2}

If the force is at an angle, we need to take the vertical component of the force (*Figure 41*).

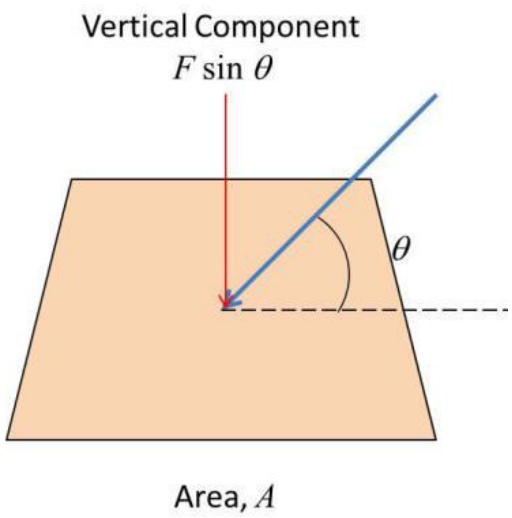


Figure 41 Force acting at an angle

So, our equation is changed to:

$$p = \frac{F \sin\theta}{A}$$

.... *Equation 23*

Pressure in a fluid **acts equally** in all directions (*Figure 42*). This is called **Pascal's Law**. This is why you are not crushed by atmospheric pressure.

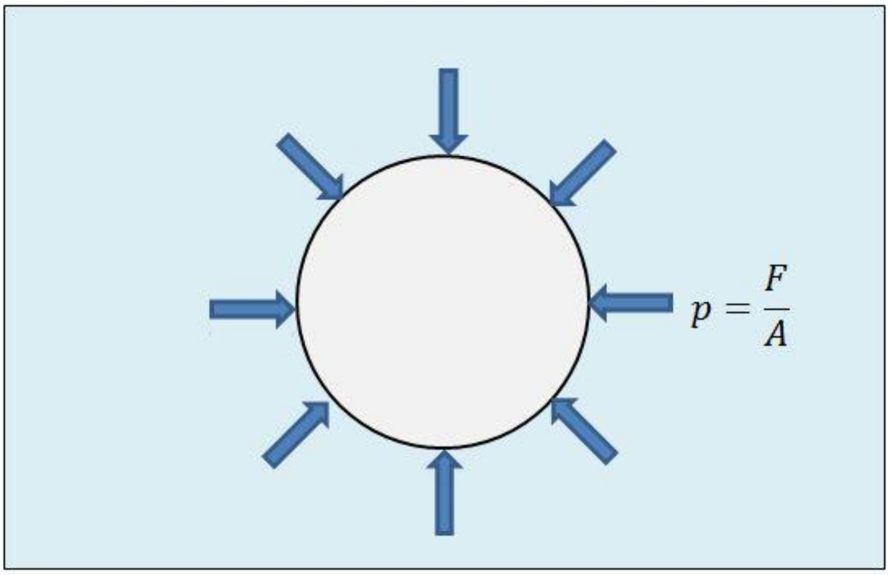


Figure 42 Pressure in a liquid acts equally in all directions

6.043 Pressure and Hydraulic Systems

This has important implications in **hydraulic systems**. Consider a very simple hydraulic system consisting of a master cylinder of area A_1 and a slave cylinder of area A_2. This is shown in the diagram below (*Figure 43*).

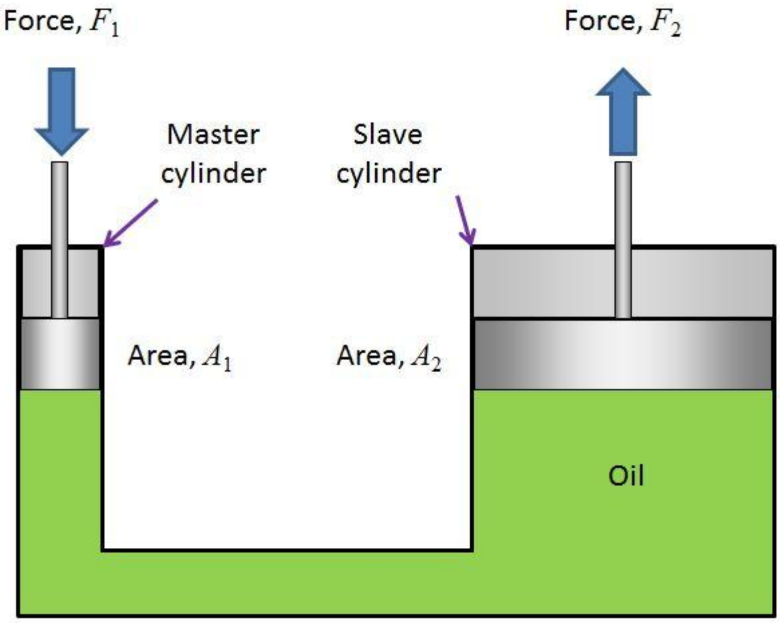


Figure 43 A simple hydraulic system

Force F_1 is applied to the master cylinder. There is a resulting pressure, p. Since pressure is the same throughout the system, pressure p acts on the slave cylinder to give force F_2. So, we can write:

$$\frac{F_1}{A_1} = p = \frac{F_2}{A_2}$$ .... *Equation 24*

Simple rearrangement gives:

$$F_1 A_2 = F_2 A_1$$ .... *Equation 25*

The system above is a **force multiplier**. The ratio of the forces = ratio of the areas. This is shown below:

$$\frac{A_2}{A_1} = \frac{F_2}{F_1}$$ .... *Equation 26*

The relationships above is an application of **Pascal's Principle**.

6.044 Pressure and Depth

SCUBA divers will tell you how when they dive, the pressure acting on them from the water increases. (SCUBA = Self Contained Underwater Breathing Apparatus).

Consider a cylinder of area A m^2, filled with fluid of density ρ kg m^{-3} to a height of h m (*Figure 44*).

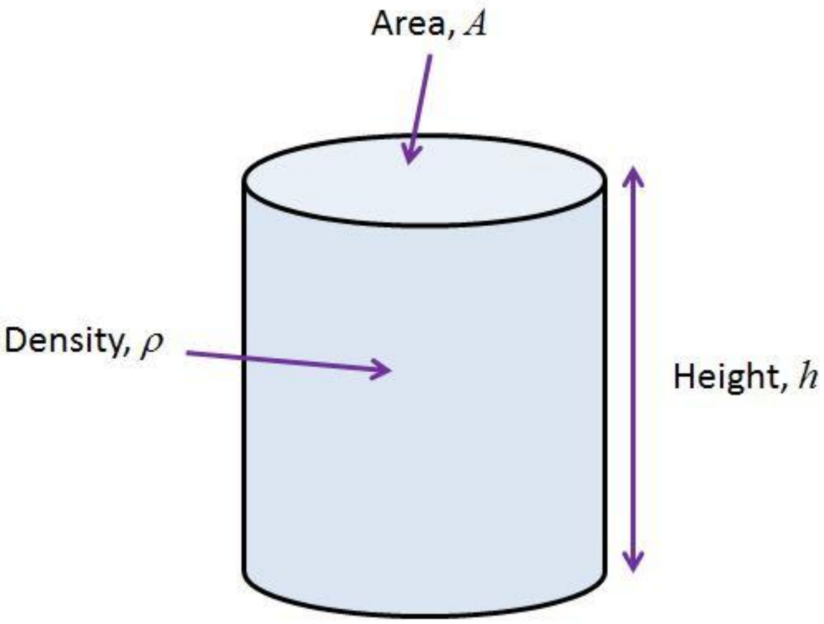


Figure 44 A cylinder of area A, height h, full of liquid of density ρ

The volume of the cylinder = area × height = hA

The mass of the fluid in the cylinder = volume × density = $hA\rho$

Remember that weight is a force, NOT the mass.

The weight of the fluid = mass × gravitational field strength = $hA\rho g$

Pressure = weight (force) ÷ area:

$$p = \frac{F}{A} = \frac{hA\rho g}{A}$$ .... *Equation 27*

The areas cancel out to give:

$$p = h\rho g$$ .... *Equation 28*

Therefore, the pressure is **proportional** to the depth in a liquid. This equation works for liquids, since they are **incompressible**, so the density of a liquid remains the same throughout the body, however deep we go. However, this model does not work for a gas. Gases are easily compressed, so the density of a gas changes with the height, something that pilots of aeroplanes need to be aware of.

The Pascal is quite a small unit of pressure. Engineers tend to use the atmospheric pressure unit, **bar**.

$$1.0 \text{ bar} = 1.0 \times 10^5 \text{ Pa}$$

Your answer to 6.04.5 should show that the pressure acting on the SCUBA diver is about 2 bar.

6.045 Archimedes' Principle (Welsh Board)

Any object that floats in a liquid has an upthrust that is equal to its weight. Since the weight acts vertically downwards, the upthrust acts vertically upwards. The idea is shown below (*Figure 45*).

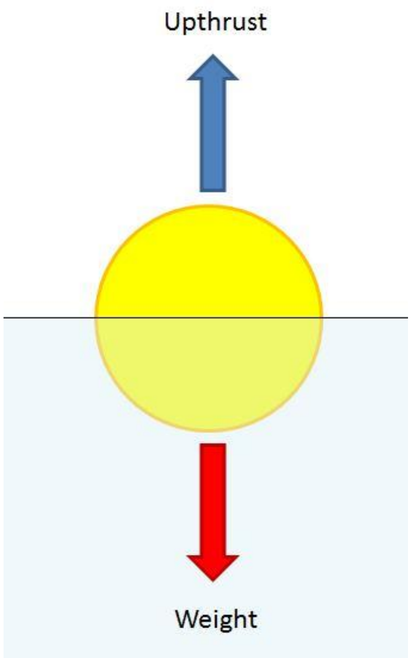


Figure 45 When an object floats in a liquid, the upthrust = weight

The weight of water displaced is the same as the weight of the object. If the upthrust is less than the weight, the object will sink.

When an object is totally immersed in the water, the volume of water displaced is the volume of the object. You will have used this idea to find out the density of an irregular object.

Archimedes' principle states:

Any body wholly or partly immersed in a fluid experiences an upthrust equal to the weight of the fluid displaced

We can explain Archimedes' Principle using the relationship between pressure and depth. Consider a uniform object of area A and thickness t at a depth of d in a liquid of density ρ (*Figure 46*).

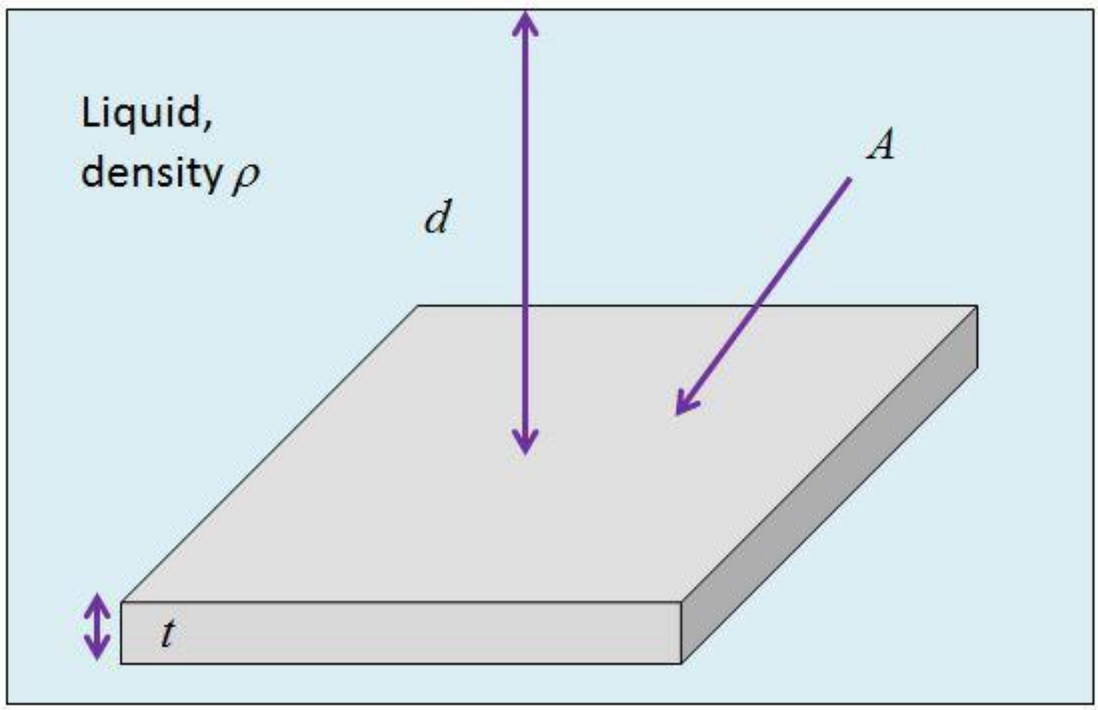


Figure 46 Explaining Archimedes' Principle

The pressure on the top surface is:

$$p_t = d\rho g$$

.........*Equation 29*

Since force = pressure × area, we can work out the downwards force on the top surface:

$$F_{\mathrm{t}} = d\rho g A$$

.... *Equation 30*

Pressure on the bottom surface is:

$$p_{\mathrm{b}} = (d + t)\rho g$$

.... *Equation 31*

Since force = pressure × area, we can work out the upwards force on the bottom surface:

$$F_{\mathrm{b}} = (d + t)\rho g A$$

............*Equation 32*

The upwards force on the object is in the opposite direction to the downwards force, therefore:

$$F_{\mathrm{r}} = (d + t)\rho g A - d\rho g A = t\rho g A$$

.... *Equation 33*

The volume of the object is tA. This volume is the same as the volume of liquid that is displaced. The mass of liquid displaced is volume × density, so the mass of liquid displaced is $tA\rho$. Therefore, the weight displaced is $tA\rho g$. We have seen how the depth term has cancelled out. This is consistent with the density of the water being the same whatever the depth, due to liquids being incompressible.

Archimedes' principle can be applied to gases. However, the upthrust from the air is small enough to be negligible.

6.046 Law of Flotation

This is a special application of the **Archimedes Principle**. Suppose we have a block of iron which has a density of 7900 kg m^{-3}. If placed in water of density 1000 kg m^{-3}, it will, of course, sink. The sunken block will displace its own volume in the water. When iron ships first started to be built, many thought they would sink like a stone. But they didn't.

Worked Example A barge has a mass of 750 tonnes (1 tonne = 1000 kg). (a) What is the volume of steel used in its construction? (Density of steel is 7800 kg m^{-3}) (b) The barge is built so that it is approximately a box 30 m long, 10 m wide, and 5 m high. Calculate the volume of the box. (c) What is the weight of water this barge will displace? Hence what volume of water will it displace? (g = 9.8 N kg^{-1}) (d) What depth will the barge float at?
Answer (a) Volume = mass ÷ density = 750 000 kg ÷ 7800 kg m^{-3} = 96.2 m^3. (b) Volume of barge = 30 m × 10 m × 5 m = 1500 m^3. (c) Weight of water = 750 000 kg × 9.8 N kg^{-1} = 7.35 × 10^6 N. (Who was going to write 750 000 kg?) The mass of water displaced is 750 000 kg. Since the density of water is 1000 kg m^{-3}, the volume is 750 m^3. (d) The area of the bottom of the barge is 300 m^2. Therefore, the depth is 750 m^3 ÷ 300 m^2 = 2.5 m. This depth is called the **draught** of the barge.

The barge will float like this (*Figure 47*).

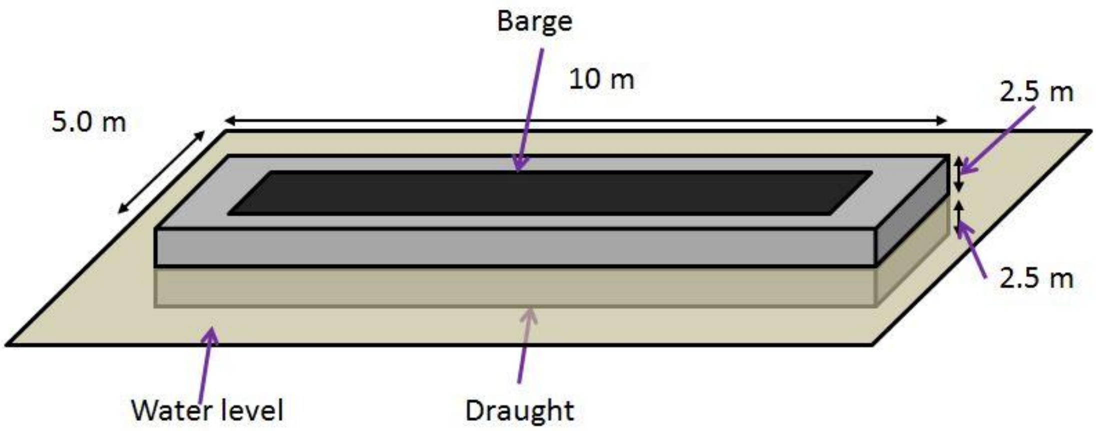


Figure 47 A barge floating on water

If the barge went out to sea, where the density of sea water is about 1050 kg m^{-3}, the draught would be reduced, and the barge would float higher in the water.

The way floating objects float higher in liquids of higher density is put to use in the **hydrometer**. This is a glass tube with a weight at the bottom, as shown (*Figure 48*).

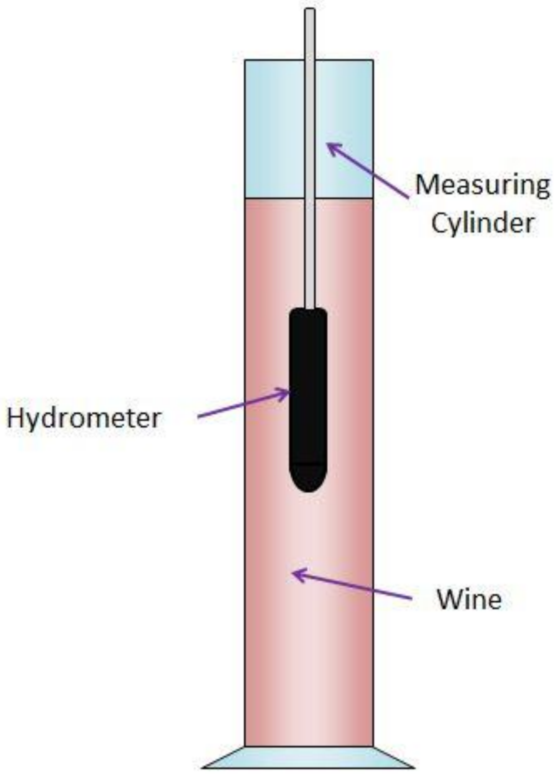


Figure 48 Hydrometer floating in a measuring cylinder of wine.

It is the home winemaker's best friend. Water with sugar dissolved has a higher density. The more sugar dissolved, the higher the density. A typical home-made wine will have enough sugar dissolved in it to give it a density of 1085 kg m^{-3} (1.085 g cm^{-3}). The hydrometer floats high in the unfermented wine. As the yeast feeds on the sugar, converting it to carbon dioxide and alcohol, the density gets lower, so the hydrometer floats lower. When the density reaches 1.000 (g cm^{-3}), the wine is considered to be fermented out and can be transferred to glass jars called demijohns or jorums to mature.

6.047 Terminal Velocity in Liquids

It is easy to measure terminal velocity in a liquid, since the terminal speed is low. This experiment is one of the core practicals (*Figure 49*).

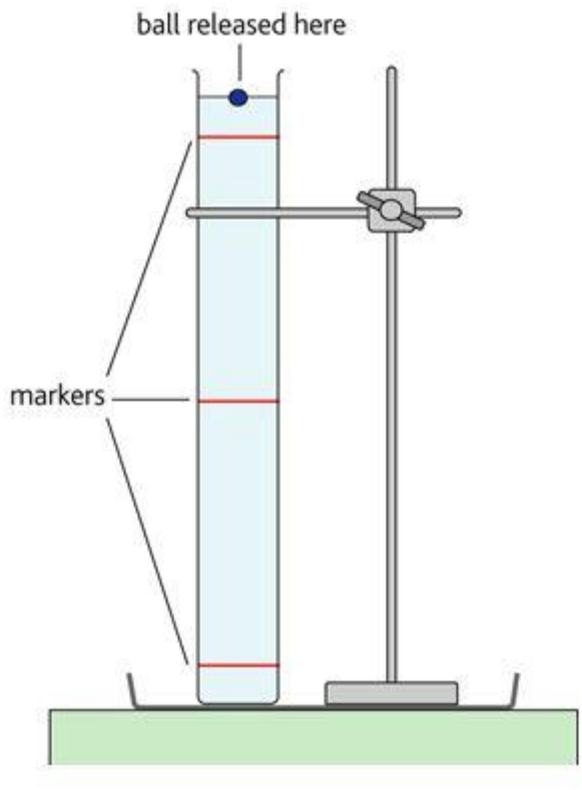


Figure 49 Measuring terminal velocity in a liquid.

When a ball bearing is dropped into a **viscous** liquid, it almost immediately reaches its terminal speed, and measuring it is simply a matter of timing the motion between two fixed points a known distance apart. You will also have to:

- Note the mass, m, of each ball.
- Use a micrometer to measure the diameter, d.

When the ball bearing drops into the liquid (usually glycerine), there are two forces:

- the weight.
- the upthrust.

Since the weight is greater than the upthrust, the ball bearing will accelerate. The velocity-time graph will look like this (*Figure 50*)

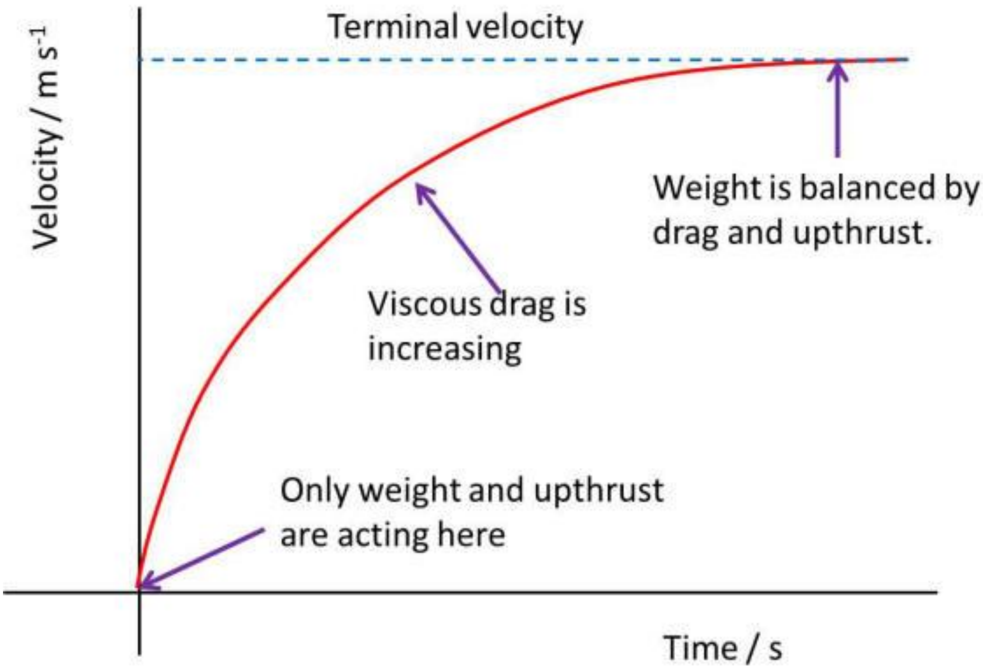


Figure 50 Velocity time graph of a ball bearing falling in a liquid.

At terminal speed, the upwards forces of **upthrust** and **drag** and the downwards force of the **weight** are **balanced** (*Figure 51*).

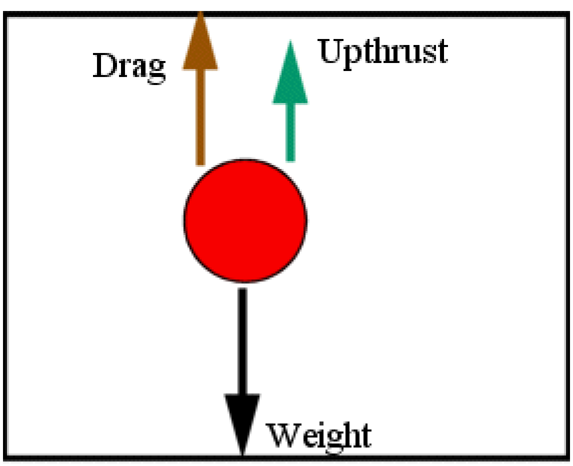


Figure 51 Forces acting on a ball bearing falling through a liquid at terminal velocity

The upthrust is the same as the weight of fluid displaced by **Archimedes' principle**. Therefore, if the weight is greater than the upthrust, the object will accelerate downwards until the drag balances the difference between the weight and the upthrust. This is true of all fluids, for example air, or water, or chocolate.

In question 6.04.7 you will see that the initial acceleration is not 9.81 m s^{-2}. This is because upthrust is a force acting in the opposite direction to the weight. However, you would not be able to time the ball bearing as it fell through the glycerine if its acceleration were 8 m s^{-2}. Some other force is acting. This is the **drag force**.

The rest of this tutorial considers the theory behind the terminal speed of a ball bearing falling through a viscous liquid like glycerine.

6.048 Viscosity and Drag Force

Viscosity has a working definition of:

the quantity that describes a fluid's resistance to flow.

There are more formal definitions, but we won't use them here. Viscosity is temperature dependent. If you heat up a viscous material like oil, it becomes less viscous, i.e. more runny. Cold car engines use more petrol because the oil is more viscous, so more energy is needed to circulate it.

When the ball bearing is falling at terminal velocity, the resultant force = 0 (Newton I). The weight is balanced by the upthrust and the drag force.

Weight - (Upthrust + Drag) = 0

6.049 Stokes' Law

We can calculate the drag force by **Stokes' Law**. It was worked out by George Gabriel Stokes (1819 - 1903). Consider a ball falling through a viscous liquid at terminal velocity. There are the three forces acting on it:

- The downwards force of weight, which is balanced by the drag and the upthrust.
- The upwards force of upthrust which can be worked out using Archimedes.
- The upwards force of drag, which is worked out by Stokes' Law.

Stokes' Law is given by:

$$F_d = 6\pi\eta r v$$

.... *Equation 34*

[F_d - drag force (N); r - radius of the sphere (m); v - terminal speed (m s^{-1}).]

The strange looking symbol, η, is "eta", a Greek lower-case letter long 'ē', the Physics Code for the **coefficient of the viscosity of a fluid**.

The units for η are N s m^{-2}. An alternative SI unit is **Pascal seconds** (Pa s).

For air, $\eta = 1.8 \times 10^{-5}$ N s m^{-2}. For glycerine, $\eta = 0.950$ N s m^{-2}.

This relationship only works when the fluid flow around the object is **laminar** (smooth). It does not work if the flow is **turbulent**.

When you are measuring the terminal velocity of ball bearings in a viscous fluid, you need to have a start point some distance below the surface of the glycerine.

So, the start line a couple of centimetres below the surface will ensure that the ball bearing is travelling at the terminal velocity. However, the Stokes' Law equation does not lend itself easily to graphical analysis. Let's look at this more closely:

$$v = \frac{F_{\mathrm{d}}}{6\pi\eta r}$$ .... *Equation 35*

The temptation is to say that:

$$v \propto \frac{1}{r}$$ .... *Equation 36*

There is a problem in that the drag force is not constant. If we use a ball bearing of half the diameter, the weight goes down by 8 times. Using the answers to the questions above, the weight of a ball bearing of diameter 1.25 mm will give these results:

Diameter / × 10^{-3} m	***Weight / N***	***Upthrust / N***	***Drag Force / N***
2.50	6.46 × 10^{-4} N	1.01 × 10^{-4} N	5.45 × 10^{-4} N
1.25	8.08 × 10^{-5} N	1.27 × 10^{-5} N	6.81 × 10^{-5} N

Therefore, the drag force will be **eight times** less. So instead of the terminal velocity being **doubled** when the radius is halved, it is **reduced by four times**. Let's look at this further.

The $6\pi\eta$ is a constant, which we will call k. So, the relationship between drag force and terminal velocity becomes:

$$v = \frac{F_{\mathrm{d}}}{kr}$$ .... *Equation 37*

If the radius is halved, the force goes down eight times. So, the new terminal velocity v' is given by:

$$v' = \frac{(F_d/8)}{kr/2} = \frac{2F_d}{8kr} = \frac{1}{4}\frac{F_d}{kr} = \frac{v}{4}$$ .... *Equation 38*

Therefore, we need to look at a new relationship between the radius and the terminal velocity.

6.0410 Relationship between Radius and Terminal Velocity

I have written two arguments here to establish the relationship between the radius of a ball bearing and its terminal velocity. The first does not take into account the upthrust (which is not on the syllabus for some boards). The second does take into account the upthrust.

Argument without upthrust

Upthrust is not on the syllabus for some of the boards. The derivation of this equation is an **approximation**, as we are **not taking into account the upthrust**. The resulting uncertainty is relatively low, compared with the uncertainties of the other measurements made in the experiment. If you want to see the effect of upthrust on the argument, please go to the next section.

Consider a ball bearing of mass m made of material of density *r* being dropped into a viscous fluid of viscosity *h*. The gravity field is *g* (= 9.81 m s^{-2} as you aren't likely to take the apparatus elsewhere).

Weight = density × volume × g

We can write this as:

$$W = \frac{4}{3}\pi r^3 \rho g$$ .... *Equation 39*

So, we can bring in the Stokes' Law equation (*Equation 34*) in by writing:

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v$$

.... *Equation 40*

Cancelling out gives us:

$$\frac{4}{3}\cancel{\pi} r^{\cancel{3}\,2} \rho g = 6\cancel{\pi}\eta \cancel{r} v$$

.... *Equation 41*

And rearranging gives:

$$r^2 = \frac{9\eta v}{2\rho g}$$

.... *Equation 42*

Then we take the square root to show the equation in the form it's usually presented:

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

.... *Equation 43*

We can rearrange to make v the subject:

$$v = \frac{2\rho g r^2}{9\eta}$$

.... *Equation 44*

Therefore, we can plot a graph of v against r^2 (*Figure 52*).

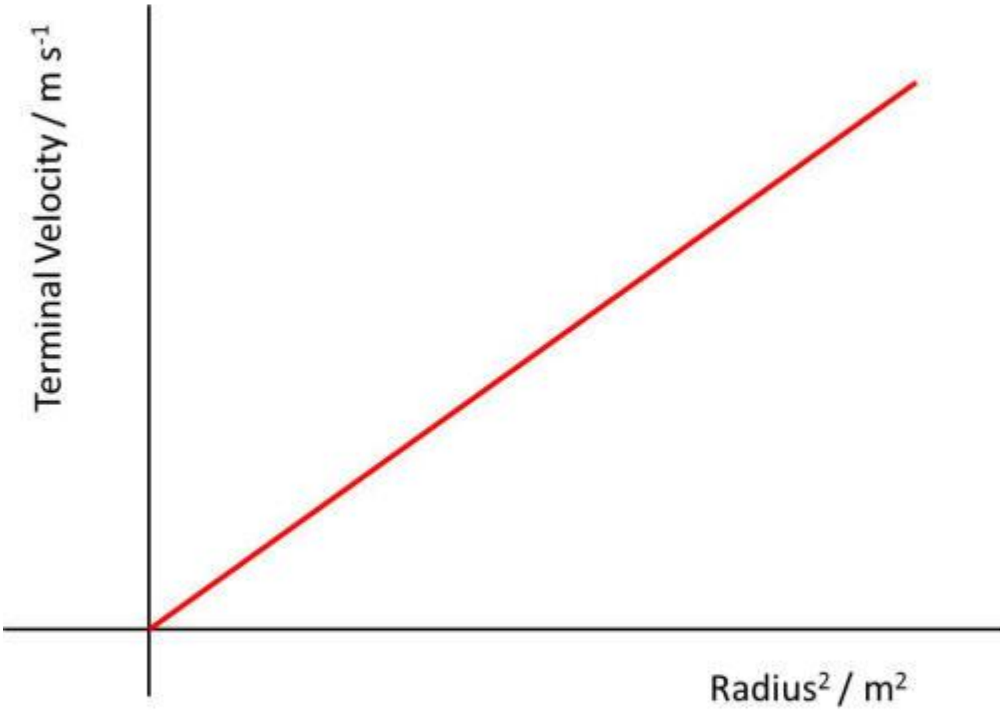


Figure 52 Graph that shows proportionality between terminal speed and square of radius.

The gradient will be:

$$m = \frac{2\rho g}{9\eta}$$

.... *Equation 45*

The term m represents the gradient.

So, if we know the density of the ball-bearing, we can work out the coefficient of the viscosity.

Argument with upthrust

The derivation of this equation takes into account the **upthrust**.

Consider a ball bearing of mass m made of material of density ρ being dropped into a viscous fluid of viscosity η, and density σ. The gravity field is g (= 9.81 m s^{-2} as you aren't likely to take the apparatus elsewhere). By Archimedes, the upthrust is the weight of the fluid displaced = volume of the object × density of the fluid × g

The strange looking symbol σ is sigma, a Greek lower-case letter 's'.

- Weight = $\rho \times V \times g$
- Upthrust $= \sigma \times V \times g$

We can write the weight as:

$$W = \frac{4}{3}\pi r^3 \rho g$$ .... *Equation 46*

We can write the upthrust as:

$$F_{\mathrm{up}} = \frac{4}{3}\pi r^3 \sigma g$$

.... *Equation 47*

We know that the drag force = weight - upthrust:

$$F_{\mathrm{d}} = W - F_{\mathrm{up}}$$

.... *Equation 48*

We can rewrite this by substituting:

$$F_d = \left(\frac{4}{3}\pi r^3 \rho g\right) - \left(\frac{4}{3}\pi r^3 \sigma g\right)$$

.... *Equation 49*

We can write this as:

$$F_d = \left(\frac{4}{3}\pi r^3 g\right)(\rho - \sigma)$$

.... *Equation 50*

So, we can bring in the Stokes' Law equation in by writing:

$$\left(\frac{4}{3}\pi r^3 g\right)(\rho - \sigma) = 6\pi\eta r v$$

.... *Equation 51*

Cancelling out gives us:

$$\left(\frac{4}{3}\cancel{\pi} r^{\overset{2}{\cancel{3}}} g\right)(\rho - \sigma) = 6\cancel{\pi}\eta\cancel{r} v$$

.... *Equation 52*

And rearranging gives:

$$r^2 = \frac{9\eta v}{2(\rho - \sigma)g}$$

.... *Equation 53*

The usual form of this equation is the square root:

$$r = \sqrt{\left(\frac{9\eta v}{2(\rho - \sigma)g}\right)}$$

.... *Equation 54*

This rearranges to make v the subject:

$$v = \frac{2(\rho - \sigma)gr^2}{9\eta}$$

.... *Equation 55*

Therefore, we can plot a graph of v against r^2 (*Figure 53*).

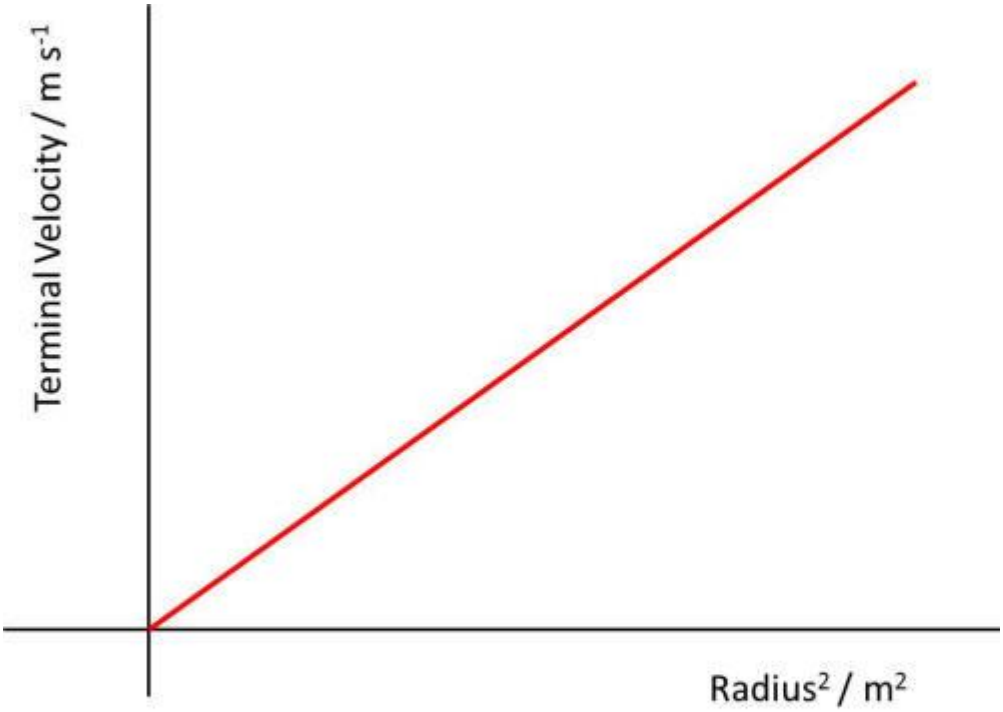


Figure 53 Graph that shows proportionality between terminal speed and square of radius

The gradient m will be:

$$m = \frac{2(\rho - \sigma)}{9\eta}$$

.... *Equation 56*

So, if we know the density of the ball-bearing and the viscous fluid, we can work out the coefficient of the viscosity. The term (ρ - σ) is simply the difference between the densities of the solid ball bearing and the viscous liquid.

The study of fluid flow is not easy and is part of university level study at second year.

Tutorial 6.04 Questions

6.04.1

Explain why glass can be considered as a super-cooled liquid.

6.04.2

A force of 250 N is applied at an angle of 40° to the horizontal onto a surface of 25 cm in diameter.

Calculate the pressure. Give your answer to an appropriate number of significant figures.

6.04.3

A simple hydraulic system has a master cylinder of area 0.0015 m^2, while the slave cylinder has an area of 0.055 m^2.

A force of 12 N is applied to the master cylinder. Calculate the force from the slave cylinder.

6.04.4

Show that your answer to Question 6.04.3 is consistent with the statement about force multipliers.

6.04.5

A SCUBA diver dives to a depth of 20 m. Calculate the pressure acting on the diver at that depth.

Density of seawater = 1030 kg m^{-3}.

Gravitational field strength = 9.81 N kg^{-1}.

6.04.6

A diver is salvaging a spherical cannon ball from the wreck of an ancient ship. The cannon ball has a diameter of 10 cm and is made of iron of density 7900 kg m^{-3}. The density of seawater is 1030 kg m^{-3}.

What is the force needed to lift the cannon ball:

(a) on land.

(b) under the water?

Does the depth of the wreck matter?

Acceleration due to gravity = 9.81 m s^{-2}.

6.04.7

A ball bearing of diameter 2.50 mm is made of steel. It is released into glycerine and falls.

(a) What is the upthrust?

(b) Show that the acceleration is about 8 ms^{-2}.

The density of this grade of steel is 8050 kg m^{-3}.

The density of glycerine is 1260 kg m^{-3}.

Acceleration due to gravity is 9.81 m s^{-2}.

6.04.8

Use answers from Question 6.04.7 to work out what the drag force is at terminal velocity. What do you notice?

6.04.9

Use your answer to Question 6.04.8 to calculate the terminal velocity of the ball bearing in Question 7, as it falls through the glycerine.

Comment on your answer.

For glycerine, η = 0.950 N s m^{-2}.

6.04.10

A student carelessly omits the upthrust in answering Questions 6.04.7, 6.04.8 and 6.04.9. What is the effect on the answer for the terminal velocity?

6.04.11

Use your answer to Question 6.04.9 to give an estimate of the distance travelled by the ball bearing as it accelerates to terminal velocity. What is your assumption?

6.04.12

A ball bearing of diameter 2.50 mm is made of steel. It is released into glycerine and falls.

Calculate the terminal velocity.

The density of this grade of steel is 8050 kg m^{-3}.

For glycerine, η = 0.950 N s m^{-2}.

Acceleration due to gravity is 9.81 m s^{-2}.

6.04.13

A ball bearing of diameter 2.50 mm is made of steel. It is released into glycerine and falls.

Calculate the terminal velocity.

The density of this grade of steel is 8050 kg m^{-3}.

The density of glycerine is 1260 kg m^{-3}.

For glycerine, h = 0.950 N s m^{-2}.

Acceleration due to gravity is 9.81 m s^{-2}.

Answers to Questions

Tutorial 6.01

6.01.1

Give one example of a material that is:

- Stiff: steel
- Brittle: glass
- Plastic: putty
- Elastic and strong: aluminium

6.01.2

1.29 g cm^{-3} = 1290 kg m^{-3}

7.6 g cm^{-3} = 7600 kg m^{-3}

19.6 g cm^{-3} = 19 600 kg m^{-3}

6.01.3

Convert 100 g to kg = 0.100 kg

$V = m/\rho$ = 0.100 kg ÷ 13 600 kg m^{-3} = **7.35 × 10^{-6} m^{3}**

6.01.4

Plastic is a name given to petrochemical polymers like polystyrene. When heated they behave plastically but at normal temperatures they are elastic.

6.01.5

a. Mass of the paint = 6.50 kg – 0.22 kg = **6.28 kg**

b. Height of paint = 0.120 m - 7.00 × 10^{-3} m = 0.113 m

Volume $= \pi D^2\ h/4$ = (π × (0.15 m)^2 × 0.113 m) ÷ 4 = **2.00 × 10^{-3} m^{3}**

c. Density = 6.28 kg ÷ 2.00 × 10^{-3} m^{-3} = 3144 kg m^{-3} = **3100 kg m^{-3} (2 s.f.)**

6.01.6

a.

Volume of aluminium = $0.60 \times 1.8 \times 10^{-4}$ m^3 = 1.08×10^{-4} m^3

Volume of magnesium = $0.4 \times 1.8 \times 10^{-4}$ m^3 = 0.72×10^{-4} m^3

Mass of aluminium = 1.08×10^{-4} m^3 × 2700 kg m^{-3} = **0.29 kg**

Mass of magnesium = 0.72×10^{-4} m^3 × 1700 kg m^{-3} = **0.12 kg**

b.

Total mass = 0.414 kg

Density = 0.414 kg ÷ 1.8×10^{-4} m^3 = **2300 kg m^{-3}**

Tutorial 6.02

6.02.1

F = 4.9 N; e = 0.12 m

k = 4.9 N ÷ 0.12 m = **41 N m^{-1}**

6.02.2

Each stretches 0.12 m, therefore both stretch 0.24 m

k = 4.9 N ÷ 0.24 m = **20.4 N m^{-1}**.

6.02.3

Each spring stretches by 6 cm = 0.06 m

New spring constant = 4.9 N ÷ 0.06 m = **82 N m^{-1}**

6.02.4

Mass on each spring = 1600 kg ÷ 4 = 400 kg

Weight on each spring = 400 kg × 9.8 N kg^{-1} = 3920 N

Compression = 0.05 m

Spring constant = 3920 N ÷ 0.05 m = **78400 N m^{-1}**

Did you forget to divide by 4?

6.02.5

$E = ½\,Fe$ and $F = ke$

$E = ½\,kee = ½\,ke^2$

6.02.6

$E = ½\ Fe$ and $e = F/k$

$E = ½\ FF/k = ½\ F^2/k$

6.02.7

Spring constant = 3920 N ÷ 0.05 m = 78400 N m^{-1} (see 6.02.4).

$E = 1/2\ ke^2$ = 0.5 × 78400 N m^{-1} × $(0.05\ m)^2$

E = **98 J**

Did you forget to divide by 4?

6.02.8

a.

$$x = \frac{F}{k} = \frac{0.300\ \text{kg} \times 9.8\ \text{N kg}^{-1}}{35\ \text{N m}^{-1}} = 0.084\ \text{m}$$

b.

F = 35 N m^{-1} × 0.094 m = **3.29 N**

c.

$$E = \frac{1}{2} \times 3.29\ \text{N} \times 0.010\ \text{m} = 0.01645\ \text{J}$$

E = 0.016 J (2 s.f.)

Tutorial 6.03

6.03.1

$$A = \pi d^2/4 = \frac{\pi \times (0.75 \times 10^{-3}\ \text{m})^2}{4} = \mathbf{4.42 \times 10^{-7}\ m^2}$$

6.03.2

Strain = 2×10^{-3} m ÷ 1.5 m = **0.0013** = 0.13 %

6.03.3

First, we need to work out the area:

$$A = \pi r^2 = \pi \times (0.5 \times 10^{-3}\ \text{m})^2 = 7.85 \times 10^{-7}\ \text{m}^2$$

Stress = F/A = 500 N ÷ 7.85×10^{-7} m² = 6.37×10^{8} Pa

Strain = e/l = 0.0080 m ÷ 2.5 m = 0.00320

Young's Modulus = stress/strain = 6.37×10^{8} Pa ÷ 0.00320 = 1.99×10^{11} Pa
= **2.0×10^{11} Pa** (2 s.f.)

6.03.4

$A = 7.85 \times 10^{-7}\ \text{m}^2$; $l = 2.5$ m; $F = 500$ N; $\Delta l = 8.0 \times 10^{-3}$ m

Formula:

$$E' = \frac{1}{2}\frac{F\Delta l}{Al}$$

Strain energy per unit volume = 0.5 × 500 N × 8.0×10^{-3} m ÷ (7.85×10^{-7} m² × 2.5 m)

E' = **1.02×10^{6} J m^{-3}**

Tutorial 6.04

6.04.1

There is no long-range crystal structure in glass. Instead, it is amorphous.

The very large groups of atoms can slide past each other, although the process take place over a very long time.

6.04.2

Formula:

$$p = \frac{F \sin \theta}{A}$$

Area = [π × (0.25 m)^2] ÷ 4 = 0.196 m^2

Pressure = (250 N × sin 40°) ÷ 0.196 m^2 = 818 Pa = **820 Pa** (2 s.f.)

6.04.3

Formula:

$$F_2 = \frac{F_1 A_2}{A_1}$$

F_2 = (12 N × 0.055 m^2) ÷ 0.0015 m^2 = **440 N**

6.04.4

Formula:

$$\frac{A_2}{A_1} = \frac{F_2}{F_1}$$

$F_2 \div F_1$ = 440 N ÷ 12 N = **36.7**

$A_2 \div A_1$ = 0.055 m^2 ÷ 0.0015 m^2 = **36.7**

The two ratios are consistent

6.04.5

Formula:

$$p = h\rho g$$

p = 20 m × 1030 kg m^{-3} × 9.8 m s^{-2} = 2.02 × 10^5 Pa = **2.0 × 10^5 Pa** (2 s.f.)

6.04.6

Volume of the cannon ball:

$$V = \frac{4}{3}\pi r^3$$

$V = 4/3 \times \pi$ × (0.050 m)^3 = 5.26 × 10^{-4} m^3

(You remembered to use the radius, not the diameter, didn't you?)

m = 5.26 × 10^{-4} m^3 × 7900 kg m^{-3} = 4.14 kg

(a)

The weight = 4.14 kg × 9.81 m s^{-2} = 40.6 N

(b)

Volume of water displaced = 5.26 × 10^{-4} m^3

Mass of water = 5.26 × 10^{-4} m^3 × 1030 kg m^{-3} = 0.542 kg

Weight of water = 0.542 kg × 9.81 m s^{-2} = 5.31 N

Force needed = weight - upthrust = 40.6 N - 5.31 N = **35.3 N** = 35 N

The depth does not matter as the density of the water is constant at any depth, as water is incompressible. The weight of water displaced will be the same, so the upthrust would be the same.

6.04.7

(a)

Volume of the ball bearing:

$$V = \frac{4}{3}\pi r^3$$

$V = 4/3 \times \pi \times (1.25 \times 10^{-3}\ \text{m})^3 = 8.18 \times 10^{-9}\ \text{m}^3$

(You remembered to use the radius, not the diameter, didn't you?)

$m = 8.18 \times 10^{-9}\ \text{m}^3 \times 8050\ \text{kg m}^{-3} = 6.59 \times 10^{-5}\ \text{kg}$

The weight = $6.59 \times 10^{-5}\ \text{kg} \times 9.81\ \text{m s}^{-2} = 6.46 \times 10^{-4}\ \text{N}$

Volume of glycerine displaced = $8.18 \times 10^{-9}\ \text{m}^3$

Mass of glycerine = $8.18 \times 10^{-9}\ \text{m}^3 \times 1260\ \text{kg m}^{-3} = 1.03 \times 10^{-5}\ \text{kg}$

Weight of glycerine = $1.03 \times 10^{-5}\ \text{kg} \times 9.81\ \text{m s}^{-2} = 1.01 \times 10^{-4}\ \text{N}$

Resultant Force = weight - upthrust = $6.46 \times 10^{-4}\ \text{N} - 1.01 \times 10^{-4}\ \text{N}$ = **$5.45 \times 10^{-4}\ \text{N}$**

(b)

Use $a = F/m = 5.45 \times 10^{-4}\ \text{N} \div 6.59 \times 10^{-5}\ \text{kg}$ = **$8.27\ \text{m s}^{-2}$**. This is about $8\ \text{m s}^{-2}$.

6.04.8

Drag = weight - upthrust

Drag = $6.46 \times 10^{-4}\ \text{N} - 1.01 \times 10^{-4}\ \text{N}$ = **$5.45 \times 10^{-4}\ \text{N}$**

The drag is equal and opposite to the resultant force.

6.04.9

Drag = weight - upthrust

Formula:

$$v = \frac{F_d}{6\pi\eta r}$$

v = 5.45 × 10^{-4} N ÷ (6 × π × 0.950 N s m^{-2} × 1.25 × 10^{-3} m) = **0.0243 m s^{-1}**

The terminal velocity is about 2 cm s^{-1}, which makes it easy to time the ball bearing as it falls through a tall measuring cylinder

6.04.10

Drag = weight

Formula:

$$v = \frac{F_d}{6\pi\eta r}$$

v = 6.46 × 10^{-4} N ÷ (6 × π × 0.950 N s m^{-2} × 1.25 × 10^{-3} m) = **0.029 m s^{-1}**

The terminal velocity is about 3 cm s^{-1}.

6.04.11

Formula:

$$v^2 = u^2 + 2as$$

Since u = 0, the formula rearranges to:

$$s = \frac{v^2}{2a}$$

s = (0.023 m s^{-1})^2 ÷ (2 × 8.27 m s^{-2}) = 1.39 × 10^{-3} m = **1.4 × 10^{-3} m** (2 s.f.)

The assumption is that the acceleration is uniform. In reality it is not.

6.04.12

Formula:

$$v = \frac{2\rho g r^2}{9\eta}$$

v = (2 × 8050 kg m^{-3} × 9.81 m s^{-2} × (1.25 × 10^{-3} m)^2) ÷ (9 × 0.950 N s m^{-2}) = **0.029 m s^{-1}**

This is consistent with the answer to 6.04.10.

6.04.13

Formula:

$$v = \frac{2(\rho - \sigma) g r^2}{9\eta}$$

v = (2 × (8050 kg m^{-3} - 1260 kg m^{-3}) × 9.81 m s^{-2} × (1.25 × 10^{-3} m)^2) ÷ (9 × 0.950 N s m^{-2})

= (2 × 6790 kg m^{-3} × 9.81 m s^{-2} × (1.25 × 10^{-3} m)^2) ÷ (9 × 0.950 N s m^{-2}) = **0.0243 m s^{-1}**

This is consistent with the answer to 6.04.9.